1. $D_{(1,2)} f(2,0) = \nabla f(2,0)\left(\frac{1}{5}, \frac{\sqrt{2}}{5}\right)$

$$= (e^y, xe^y)\big|_{(x,y)=(2,0)}\left(\frac{1}{5}, \frac{\sqrt{2}}{5}\right) = (1,2)\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = \sqrt{5}$$

2.

Let $f = x^2+y^2-z=0$

$\nabla f\big|_{(2,-2,8)} = (f_x, f_y, f_z) = (2x, 2y, -1) = (4, -4, 1)$

Tangent plane $\Rightarrow 4x-4y-z=8$

normal line $\Rightarrow \frac{x-2}{4} = \frac{y+2}{-4} = \frac{z-8}{-1}$

3.

1° $\nabla f = \left(2xe^{1-x^2-y^2} + (x^2+3y^2)(-2x)\cdot e^{1-x^2-y^2},\ 6ye^{1-x^2-y^2} + (x^2+3y^2)(-2y)e^{1-x^2-y^2}\right)$

$$= e^{1-x^2-y^2}\left(2x(1-x^2-3y^2),\ 2y(3-x^2-3y^2)\right) = (0,0)$$

$\Rightarrow \begin{cases} 2x(1-x^2-3y^2)=0 \\ 2y(3-x^2-3y^2)=0 \end{cases}$

$\Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$ or $\begin{cases} x=0 \\ 3-x^2-3y^2=0 \end{cases}$ or $\begin{cases} 1-x^2-3y^2=0 \\ y=0 \end{cases}$

$\Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$ or $\begin{cases} x=0 \\ y=\pm1 \end{cases}$ or $\begin{cases} x=\pm1 \\ y=0 \end{cases}$

$\Rightarrow$ critical pts : $(0,0), (0,1), (0,-1), (-1,0), (1,0)$

$$\Rightarrow Hf = \begin{pmatrix} -2xe^{1-x^2-y^2}(2x-2x^3-6xy^2) + e^{1-x^2-y^2}(2-6x-6y^2) & -2xe^{1-x^2-y^2}(6y-2x^2y-6y^3) + e^{1-x^2-y^2}(-4xy) \\ -2ye^{1-x^2-y^2}(2x-2x^3-6xy^2) + e^{1-x^2-y^2}(-12xy) & -2ye^{1-x^2-y^2}(6y-2x^2y-6y^3) + e^{1-x^2-y^2}(6-2x^2-18y^2) \end{pmatrix}$$

$$= e^{1-x^2-y^2}\begin{pmatrix} -4x^2+4x^4+12x^2y^2+2-6x^2-6y^2 & -12xy+4x^3y+12xy^3-4xy \\ -4xy+4x^3y+12xy^3-12xy & -12y^2+4x^2y^2+12y^4+6-2x^2-18y^2 \end{pmatrix}$$

$$= e^{1-x^2-y^2}\begin{pmatrix} -10x^2+4x^4+12x^2y^2+2-6y^2 & -16xy+4x^3y+12xy^3 \\ -16xy+4x^3y+12xy^3 & -30y^2+4x^2y^2+12y^4+6-2x^2 \end{pmatrix}$$

2° $Hf(0,0) = e\begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 2e & 0 \\ 0 & 6 \end{pmatrix} \Rightarrow \begin{cases} f_{xx}(0,0) = 2e > 0 \\ \Delta(0,0) = 12e^2 > 0 \end{cases} \Rightarrow f(0,0)$ is a local min.

$Hf(\pm1,0) = 1\cdot\begin{pmatrix} -10+4+2 & 0 \\ 0 & 6-2 \end{pmatrix} = \begin{pmatrix} -4 & 0 \\ 0 & 4 \end{pmatrix} \Rightarrow \begin{cases} f_{xx}(\pm1,0) = -4 < 0 \\ \Delta(\pm1,0) = -16 < 0 \end{cases} \Rightarrow (\pm1, 0, f(\pm1,0))$ is a saddle pt.

$Hf(0,\pm1) = 1\cdot\begin{pmatrix} 2-6 & 0 \\ 0 & -30+12+6 \end{pmatrix} = \begin{pmatrix} -4 & 0 \\ 0 & -12 \end{pmatrix} \Rightarrow \begin{cases} f_{xx}(0,\pm1) = -4 < 0 \\ \Delta(0,\pm1) = 48 > 0 \end{cases} \Rightarrow f(0,\pm1)$ is a local max.

3° local miniman : $f(0,0) = (0^2-3\cdot0^2)e^{1-0^2-0^2} = 0$

local maximan : $f(0,\pm1) = (0^2-3(\pm1)^2)e^{1-0^2-(\pm1)^2} = -3\cdot1 = -3$

saddle pts : $(\pm1, 0, f(\pm1,0)) = (\pm1, 0, 1)$ #