1. $f(x,y) = e^{xy}$, $x(u,v) = 3u\sin v$, $y(u,v) = 4v^2u$

$\frac{\partial f}{\partial x} = ye^{xy}$, $\frac{\partial f}{\partial y} = xe^{xy}$, $\frac{\partial x}{\partial u} = 3\sin v$, $\frac{\partial y}{\partial u} = 4v^2$

$$\frac{\partial f}{\partial u} = ye^{xy}(3\sin v) + xe^{xy}(4v^2)$$
$$= 4v^2e^{12u^2v^2\sin v}(3\sin v) + 3u\sin v\, e^{12u^2v^2\sin v}(4v^2) \quad \#$$

$\frac{\partial x}{\partial v} = 3u\cos v$, $\frac{\partial y}{\partial v} = 8vu$

$$\frac{\partial f}{\partial v} = ye^{xy}(3u\cos v) + xe^{xy}(8vu)$$
$$= 4v^2ue^{12u^2v^2\sin v}(3u\cos v) + 3u\sin v\, e^{12u^2v^2\sin v}(8vu) \quad \#$$

2. (a) $f(x,y,z) = 3x^2z + 2z^3 - 3yz$

$f_x(x,y,z) = 6xz$, $f_y(x,y,z) = 3z$, $f_z(x,y,z) = 3x^2 + 6z^2 - 3y$

$$\frac{\partial z}{\partial x} = \frac{-6xz}{3x^2+6z^2-3y}$$
$$\frac{\partial z}{\partial y} = \frac{3z}{3x^2+6z^2-3y}$$

(b) $f(x,y,z) = 3e^{xyz} - 4xz^2 + x\cos y - 2$

$f_x(x,y,z) = 3yze^{xyz} - 4z^2 + \cos y$, $f_y(x,y,z) = 3xze^{xyz} - x\sin y$

$f_z(x,y,z) = 3xye^{xyz} - 8xz$

$$\frac{\partial z}{\partial x} = \frac{3yze^{xyz} - 4z^2 + \cos y}{8xz - 3xye^{xyz}} \quad \# \qquad \frac{\partial z}{\partial y} = \frac{3xze^{xyz} - x\sin y}{8xz - 3xye^{xyz}} \quad \#$$

3.

$$u = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$$

$$D_u f(x,y) = (-2xe^{-(x^2+y^2)})\frac{1}{\sqrt{2}} + (-2ye^{-(x^2+y^2)})\frac{1}{\sqrt{2}}$$

$$D_u f(0,0) = 0 \quad \#$$

4.

$$\overrightarrow{PQ} = 2i + j \quad , u = \frac{2}{\sqrt{5}} i + \frac{1}{\sqrt{5}} j$$

$$D_u f(x,y) = e^y \cos x \cdot \frac{2}{\sqrt{5}} + e^y \sin x \cdot \frac{1}{\sqrt{5}}$$

$$D_u f(0,0) = \frac{2}{\sqrt{5}} \quad \#$$