1. We know that $1 \le \sin(\frac{1}{y}) \le 1$, $-1 \le \cos(\frac{1}{x}) \le 1$

$$\Rightarrow -x^2 \le x^2\sin(\frac{1}{y}) \le x^2,\quad -y^2 \le y^2\cos(\frac{1}{x}) \le y^2$$

$$\Rightarrow -(x^2+y^2) \le x^2\sin(\frac{1}{y}) + y^2\cos(\frac{1}{x}) \le x^2+y^2$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} -(x^2+y^2) = 0 \le \lim_{(x,y)\to(0,0)} [x^2\sin(\frac{1}{y}) + y^2\cos(\frac{1}{x})]$$

$$\le \lim_{(x,y)\to(0,0)} x^2+y^2 = 0.$$

By squeeze thm, $\lim_{(x,y)\to(0,0)} [x^2\sin(\frac{1}{y}) + y^2\cos(\frac{1}{x})] = 0$.

2. $\lim_{y\to 0}\left[\lim_{x\to 0} \frac{\sin(x^2)-y^2}{x^2+y^2}\right] = \lim_{y\to 0}\left(\frac{-y^2}{y}\right) = -1$

3. $\lim_{x\to 0}\left[\lim_{y\to 0} \frac{\sin(x^2)-y^2}{x^2+y^2}\right] = \lim_{x\to 0}\frac{\sin(x^2)}{x^2} = 1$

4. Let $u = x-1$, $v = y+1$

原式 $= \lim_{(u,v)\to(0,0)} \frac{(u+1)(v-1)+1}{(u+1)^2-(v+1)^2}$, Let $u = r\cos\theta$, $v = r\sin\theta$

$$\Rightarrow \lim_{r\to 0} \frac{r^2\cos\theta\sin\theta + r\cos\theta + r\sin\theta}{r^2\cos^2\theta + 2r\cos\theta + 1 - r^2\sin^2\theta - 2r\sin\theta - 1} = \frac{\cos\theta+\sin\theta}{2(\cos\theta-\sin\theta)}$$

∴ 原式 D.N.E.

5.

$$\frac{\partial z}{\partial x} = 2xy^3 + ye^{xy}$$

$$\frac{\partial z}{\partial y} = 3x^2y^2 + xe^{xy}$$