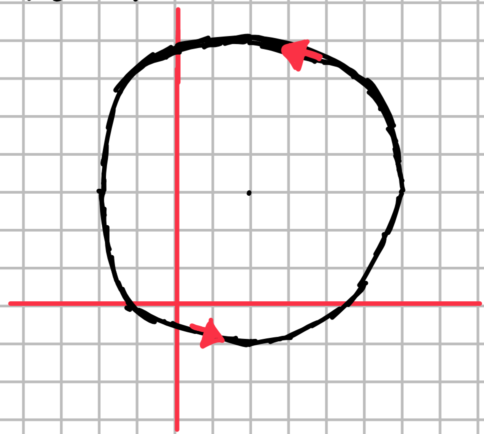
1 (a)

$$\begin{cases} x = 2 + 4\cos t \\ y = 3 + 4\sin t \end{cases} \Rightarrow \begin{cases} \cos t = \dfrac{x-2}{4} \\ \sin t = \dfrac{y-3}{4} \end{cases}$$

$$\therefore \frac{(x-2)^2}{16} + \frac{(y-3)^2}{16} = 1$$

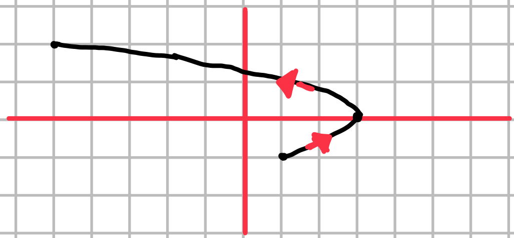
半徑為4，中心在(2,3)



(b)

$$\begin{cases} x = 6 - t^2 \\ y = \dfrac{t}{2} \end{cases}$$

$t = 2y \Rightarrow x = 6 - 4y^2$



2.
(a)

$$\frac{dx}{dt} = -2\sin t + 2\cos 2t \qquad \frac{dy}{dt} = 2\cos t - 4\sin 2t$$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\frac{dy}{dt}(0)}{\frac{dx}{dt}(0)} = \frac{2\cos 0 - 4\sin 0}{-2\sin 0 + 2\cos 0} = 1 \quad \#$$

(b)

$$\begin{cases} x = 2\cos t + \sin 2t = 0 \\ y = 2\sin t + 2\cos 2t = -3 \end{cases}$$

$\Rightarrow 2\cos t + \sin 2t = 2\cos t + 2\sin t\cos t = 2\cos t(1+\sin t) = 0$

$\Rightarrow \cos t = 0$ or $\sin t = -1$

Case: $\sin t = -1 \Rightarrow t = \frac{3\pi}{2}$

t代入:

$$\begin{cases} x = 2\cos\left(\frac{3\pi}{2}\right) + \sin(3\pi) = 0 \ ✓ \\ y = 2\sin\left(\frac{3\pi}{2}\right) + \cos(3\pi) = -4 \ ✗ \end{cases}$$

∴ 切線斜率不存在

3. $\frac{x^2}{9}+\frac{y^2}{4}=1 \Rightarrow x=3\cos t,\ y=2\sin t$ for $0\le t\le\pi$

$$S=\int_0^{\pi} 2\pi(2\sin t)\sqrt{(-3\sin t)^2+(2\cos t)^2}\,dt$$

$\sin^2 t+\cos^2 t=1$

$$=4\pi\int_0^{\pi}\sin t\sqrt{9\sin^2 t+4\cos^2 t}\,dt$$

$$=4\pi\int_0^{\pi}\sin t\sqrt{9-5\cos^2 t}\,dt=4\pi\int_0^{\pi}3\sin t\sqrt{1-\frac{5}{9}\cos^2 t}\,dt$$

令 $u=\frac{\sqrt5}{3}\cos t$, $du=-\frac{\sqrt5}{3}\sin t\,dt$

$$=4\pi\int_{-\frac{\sqrt5}{3}}^{\frac{\sqrt5}{3}}\frac{9}{\sqrt5}\sqrt{1-u^2}\,du$$

$$=4\pi\left[\frac{9}{\sqrt5}\times\frac{1}{2}\left(\sin^{-1}u+u\sqrt{1-u^2}\right)\right]_{-\frac{\sqrt5}{3}}^{\frac{\sqrt5}{3}}$$

$$=4\pi\frac{9}{2\sqrt5}\left[\left(\sin^{-1}\frac{\sqrt5}{3}+\frac{\sqrt5}{3}\sqrt{1-\frac{5}{9}}\right)-\left(\sin^{-1}\frac{-\sqrt5}{3}-\frac{\sqrt5}{3}\sqrt{1-\frac{5}{9}}\right)\right]$$

$$=\frac{18\pi}{\sqrt5}\left(\sin^{-1}\frac{\sqrt5}{3}-\sin^{-1}\frac{-\sqrt5}{3}+\frac{2\sqrt5}{3}\times\sqrt{\frac{4}{9}}\right)$$

$$=\frac{36\pi}{\sqrt5}\sin^{-1}\frac{\sqrt5}{3}+8\pi\approx 67.7$$ #