1.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{2^k}$ (use ratio test)

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\frac{k+1}{2^{\cancel{k+1}}} \times \frac{2^{\cancel{k}}}{k} = \lim_{k\to\infty}\frac{k+1}{2k} = \frac{1}{2} < 1$$

Ans: the series is converges.

(b) $\sum_{k=2}^{\infty} \frac{e^{3k}}{k^{3k}}$ (use root test)

$$\lim_{k\to\infty}\sqrt[k]{\frac{e^{3k}}{k^{3k}}} = \lim_{k\to\infty}\frac{e^3}{k^3} = 0 < 1$$

Ans: the series is converge

2.

(a) we know maclaurin polynomial for $f(x)=e^x$ is:

$$P_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

so $f(x) = xe^x$ is:

$$P_n(x) = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \cdots + \frac{x^{n+1}}{n!}$$

$$\therefore P_5(x) = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!}$$

$$P_5\left(\frac{1}{4}\right) \approx 0.3218 \#$$

(b) $f(x)=\frac{x}{x+1}$, $f(0)=$

$f(x)=\frac{x}{x+1}$	$f(0)=0$
$f'(x)=\frac{1}{(x+1)^2}$	$f'(0)=1$
$f''(x)=\frac{-2}{(x+1)^3}$	$f''(0)=-2$
$f'''(x)=\frac{6}{(x+1)^4}$	$f'''(0)=6$
$f^4(x)=\frac{-24}{(x+1)^5}$	$f^4(0)=-24$

$$P_4(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^4(0)}{4!}x^4$$
$$=x-x^2+x^3-x^4$$

$$P_4(1)=1-1+1-1=0 \#$$

3 (a) $\sum_{k=0}^{\infty} k!(x-5)^k$ (use ratio test)

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right|=\lim_{k\to\infty}\left|\frac{(k+1)!(x-5)^{k+1}}{k!(x-5)^k}\right|=\lim_{k\to\infty}\frac{(k+1)\cancel{k!}|x-5|}{\cancel{k!}}$$

$$=\lim_{k\to\infty}[(k+1)|x-5|]=\begin{cases}0 & \text{if } x=5\\ \infty & \text{if } x\neq 5\end{cases}$$

Ans: radius of convergence $=0$

(b) $\sum_{k=1}^{\infty}\frac{x^k}{k4^k}$ (use ratio test)

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right|=\lim_{k\to\infty}\left|\frac{x^{\cancel{k}+1}}{(k+1)\cancel{4^{k}}4}\cdot\frac{k\cancel{4^k}}{\cancel{x^k}}\right|=\frac{|x|}{4}\lim_{k\to\infty}\frac{k}{k+1}=\frac{|x|}{4}<1$$

$$\Rightarrow |x|<4$$

Ans: radius of convergence $=4$