1.

(a) $a_n = \frac{\sin(n)}{n^2}$

$-1 \le \sin(n) \le 1$, for all n

$\Rightarrow \frac{-1}{n^2} \le \frac{\sin(n)}{n^2} \le \frac{1}{n^2}$, for all $n \ge 1$

$\because \lim_{n\to\infty} \frac{-1}{n^2} = \frac{1}{n^2} = 0$, $\lim_{n\to\infty} \frac{\sin(n)}{n^2} = 0$

Ans: sequence $\frac{\sin(n)}{n^2}$ is converge to 0.

(b) $a_n = \frac{n^2+1}{2n-3}$

$$\lim_{n\to\infty} \frac{n^2+1}{2n-3} = \lim_{n\to\infty} \frac{n+\frac{1}{n}}{2-\frac{3}{n}} = \infty$$

Ans: sequence $\frac{n^2+1}{2n-3}$ is diverges.

(c) $a_n = \frac{n+1}{e^n}$

$\lim_{n\to\infty} \frac{n+1}{e^n}$ $\frac{\infty}{\infty}$

$$= \lim_{n\to\infty} \frac{\frac{d}{dn}(n+1)}{\frac{d}{dn}(e^n)} = \lim_{n\to\infty} \frac{1}{e^n} = 0$$

Ans: sequence $\frac{n+1}{e^n}$ is converge to 0

2.

(a) $\sum_{k=0}^{\infty} 3\left(\frac{1}{5}\right)^k$ ← Geometric series

$r=\frac{1}{5}<1$, converges

$\lim_{n\to\infty} S_n = \frac{a}{1-r} = \frac{3}{1-\frac{1}{5}} = \frac{15}{4}$

Ans: the series converges to $\frac{15}{4}$

(b)

$\sum_{k=0}^{\infty} \frac{1}{2}(3)^k$ ← Geometric series

$r=3>1$, diverge

Ans: the series is diverge.

(c)

$\sum_{k=1}^{\infty} \frac{4}{k(k+2)}$

令 $\frac{4}{k(k+2)} = \frac{A}{k} + \frac{B}{k+2} \Rightarrow (A+B)k + 2A = 4$ $\begin{cases} A=2 \\ B=-2 \end{cases}$

$\therefore \frac{4}{k(k+2)} = \frac{2}{k} - \frac{2}{k+2}$

$S_n = \sum_{k=1}^{\infty}\left(\frac{2}{k} - \frac{2}{k+2}\right) = \left(\frac{2}{1} - \cancel{\frac{2}{3}}\right) + \left(\frac{2}{2} - \cancel{\frac{2}{4}}\right) + \left(\cancel{\frac{2}{3}} - \cancel{\frac{2}{5}}\right) + \left(\cancel{\frac{2}{4}} - \cancel{\frac{2}{6}}\right)$

$+ \ldots\ldots + \left(\cancel{\frac{2}{n-1}} - \frac{2}{n+1}\right) + \left(\cancel{\frac{2}{n}} - \frac{2}{n+2}\right)$

$S_n = 3 - \frac{2}{n+1} - \frac{2}{n+2}$, $\lim_{n\to\infty} S_n = 3$

Ans: the series is converges to 3