1. (a) $\int_0^6\int_{\frac{y}{2}}^3 (x+y)\,dx\,dy = \int_0^6\left[\frac{x^2}{2}+xy\right]_{\frac{y}{2}}^3 dy$

$= \int_0^6 \frac{9}{2}+3y-\frac{5}{8}y^2\,dy = \left[\frac{9}{2}y+\frac{3}{2}y^2-\frac{5}{24}y^3\right]_0^6 = 36$ #

(b) $\int_0^4\int_0^x (4e^{x^2}-5\sin y)\,dy\,dx = \int_0^4\left[4ye^{x^2}+5\cos y\right]_0^x dx$

$= \int_0^4 4xe^{x^2}+5\cos x-5\,dx = \left[2e^{x^2}+5\sin x-5x\right]_0^4 = 2e^{16}+5\sin 4-22$ #

2. (a)

$\int_0^{2\pi}\int_0^{2-2\sin\theta} r\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^{2-2\sin\theta} d\theta$

$= \int_0^{2\pi} 2-4\sin\theta+2\sin^2\theta\,d\theta = \left[2\theta+4\cos\theta+\theta-\frac{\sin 2\theta}{2}\right]_0^{2\pi}$

$= \left[3\theta+4\cos\theta-\frac{\sin 2\theta}{2}\right]_0^{2\pi} = 6\pi$ #

(b) $\int_0^{2\pi}\int_2^3 (9-r^2)r\,dr\,d\theta = \int_0^{2\pi}\int_2^3 9r-r^3\,dr\,d\theta$

$= \int_0^{2\pi}\left[\frac{9}{2}r^2-\frac{r^4}{4}\right]_2^3 d\theta = \int_0^{2\pi}\frac{25}{4}\,d\theta = \frac{25}{2}\pi$ #

3.

$z=1+x^2+y^2$, 令 $z=5 \Rightarrow x^2+y^2=4$ 拋物面與平面交於一个半徑為2以原點為中心的圓

$S=\iint_R\sqrt{f_x^2+f_y^2+1}\,dA = \iint_R\sqrt{4x^2+4y^2+1}\,dA$

令 $x=r\cos\theta$
$y=r\sin\theta$
$x^2+y^2=r^2$
$dA=r\,dr\,d\theta$

$S=\int_0^{2\pi}\int_0^2\sqrt{4r^2+1}\cdot r\,dr\,d\theta = \frac{1}{8}\int_0^{2\pi}\left[\frac{2}{3}(4r^2+1)^{\frac{3}{2}}\right]_0^2 d\theta$

$= \frac{1}{12}\int_0^{2\pi}(17^{\frac{3}{2}}-1)\,d\theta = \frac{2\pi}{12}(17^{\frac{3}{2}}-1)$ #