1. (12%) Let $\boldsymbol{r}(\text{t}) = \frac{1}{\sqrt{t+1}}\mathbf{i} + \frac{\sin(2t)}{t}\mathbf{j} + \frac{1}{t}\mathbf{k}$, $\boldsymbol{g}(\text{t}) = \sqrt{1+t}\mathbf{i} + t\mathbf{j} + \frac{t}{t-2}\mathbf{k}$:

   (a) Evaluate the limit $\lim_{t\to 0} \boldsymbol{r}(t)$

   (b) Find the intervals on which the curve given by $\boldsymbol{g}(t)$ is smooth

   (c) Compute $\frac{\mathrm{d}}{\mathrm{dt}}[\boldsymbol{r}(t)\cdot\boldsymbol{g}(t)]$

2. (8%) Find the following limits:

   (a) $\lim_{(x,y)\to(0,0)} \frac{2x}{x^2-y^2}$

   (b) $\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)}{4}\cdot\ln(x^2+y^2)$

3. (15%)

(a) Let $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & \text{when } (x,y)\neq(0,0) \\ 0 & \text{when } (x,y)=(0,0) \end{cases}$, compute $f_x(0,0)$ and $f_y(0,0)$.

In addition, decide whether f differentiable at $(0,0)$

(b) Given the equation $xz^2 - ysin(z) = 0$, find the first–order partial derivatives $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ using implicit differentiation.

(c) Considering the level surface defined by $z^5 + (sin(x))z^3 + yz = 4$. Find an equation of the tangent plane at the point $(0,3,1)$

4. (10%) Let $f(x,y) = e^{-x}\cos(y)$

   (a) Compute the directional derivative of f at (1,0) in the direction from $P(1,0)$ to $Q(6,12)$

   (b) Find the direction in which f has minimum increase at $(1,0)$. What is the minimum rate of increase?

5. (10%) Let $f(x,y) = x^3 + y^3 - 3xy$

   (a) Find the critical points of f

   (b) Classify each critical point as a local maximum, local minimum or saddle point

6. (15%) Evaluate the following expressions

   (a) $\int_0^4 \int_{\sqrt{x}}^2 \sin(\frac{x}{y})\, dydx$

   (b) $\int_0^2 \int_0^{\sqrt{4-x^2}} \cos(\sqrt{x^2+y^2})\, dydx$

   (c) $\int_0^{\frac{\pi}{4}} \int_0^6 \int_0^{6-r} rz\ \ dzdrd\theta$

7. (10%) Find the area of the surface given by $z = f(x,y) = \frac{1}{2}y^2$ that lies above the region R where R is a square with vertices $(0,0), (3,0), (0,3), (3,3)$

8. (10%) Find the volume of the solid bounded above by $x^2 + y^2 + z^2 = 36$ and below by $z = \sqrt{x^2 + y^2}$

9. (10%) Use the change of variables to find the volume of the solid region lying below the surface $z = f(x,y) = \ln(x^2y + x)$ and above the plane region R wher R is a region bounded by $xy = 1, xy = 3, x = 1, x = e$.