1. (24%) Determine whether the series converges absolutely or conditionally, or diverges. In addition, please indicate the test you use.

(a) $\sum_{n=1}^{\infty}(-1)^{n+1}ne^{-n^2}$

(b) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(\sqrt{n})}{n^{\frac{3}{2}}}$

(c) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2n^2}{n^3+3}$

(d) $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^{n^2}$

2. (16%) Find the interval of convergence of the power series (Be sure to check the for the convergence at the endpoints of the intervals)

(a) $\sum_{n=1}^{\infty}\frac{n}{n+1}\frac{(x)^n}{(2x+1)^n}$

(b) $\sum_{n=1}^{\infty}\frac{n!(x+1)^n}{3^n}$

3. (10%) Let $f(x)=\sqrt{1+x}+\sqrt{1-x}$, what is $f^{(10)}(0)=?$

4. (18%) Evaluate the following expression (Try to use the Basic series of Taylor series and notice that the power series is a continuous function)

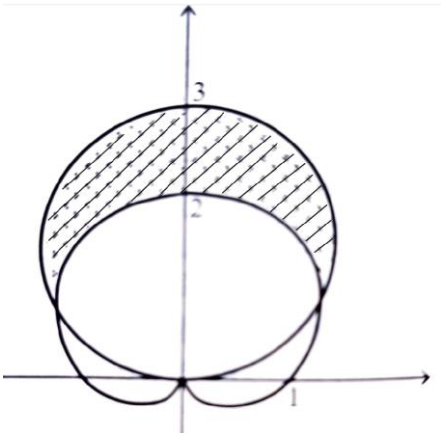
(a) $\sum_{n=0}^{\infty}\frac{3^{n+1}}{n!}$

(b) $\frac{1}{1\times2}-\frac{1}{2\times2^2}+\frac{1}{3\times2^3}-\frac{1}{4\times2^4}+\ldots$

(c) $\lim_{x\to0^+}\frac{arctan(2x)-sin(2x)}{sinx-x}$

5. (12%) Derive the Maclaurin series of $f(x) = arccot(2x)$ (Do not use the results from $arctan(x)$)

6. (10%) Find the area of the shaded region bounded by the curves $r = 1 + sin(\theta)$ and $r = 3sin(\theta)$



7. (10%) Find the area of the surface formed by revolving the polar graph $r = 2(1 + sin(\theta))$ about the $\theta = \frac{\pi}{2}$ over the interval $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$

Function	Taylor series	Interval of convergence
$\frac{1}{x}$	$1-(x-1)+(x-1)^2-(x-1)^3+\cdots.+(-1)^n(x-1)^n+\cdots.$	$0<x<2$
$\frac{1}{1+x}$	$1-x+x^2-x^3+\cdots.+(-1)^n x^n+\cdots.$	$-1<x<1$
$\ln x$	$(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\cdots.+\frac{(-1)^{n-1}(x-1)^n}{n}+\cdots.$	$0<x\leq 2$
e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots.+\frac{x^n}{n!}+\cdots.$	$-\infty<x<\infty$
$sin(x)$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots.+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots.$	$-\infty<x<\infty$
$cos(x)$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots.+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots.$	$-\infty<x<\infty$
$arctan(x)$	$x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots.+\frac{(-1)^n x^{2n+1}}{2n+1}+\cdots.$	$-1\leq x\leq 1$
$arcsin(x)$	$x+\frac{x^3}{2\times 3}+\frac{1\times 3x^5}{2\times 4\times 5}+\cdots.+\frac{(2n)!\,x^{2n+1}}{(2^n n!)^2(2n+1)}+\cdots.$	$-1\leq x\leq 1$
$(1+x)^k$	$1+kx+\frac{k(k-1)x^2}{2!}+\frac{k(k-1)(k-2)x^3}{3!}+\cdots.+\frac{k(k-1)\ldots(k-n+1)x^n}{n!}+\cdots.$	$-1<x<1$