1.

$$\sum_{n=1}^{\infty}\frac{(n!)^n}{(n^n)^2}=\sum_{n=1}^{\infty}\frac{(n!)^n}{(n^2)^n}$$

$$\lim_{n\to\infty}\sqrt[n]{|a_n|}=\lim_{n\to\infty}\sqrt[n]{\frac{(n!)}{(n^2)^n}}=\lim_{n\to\infty}\frac{n!}{n^2}=\infty$$

Therefore, by the Root Test, the series diverges.

2.

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|=\lim_{n\to\infty}\left|\frac{(x-3)^{n+2/[(n+2)4^{n+2}]}}{(x-3)^{n+1/[(n+1)4^{n+1}]}}\right|=\lim_{n\to\infty}\left|\frac{(x-3)(n+1)}{4(n+2)}\right|=\lim_{n\to\infty}\left|\frac{x-3}{4}\right|$$

$R=4$

Interval : $\quad -1<x<7$

When $x=7$, $\displaystyle\sum_{n=0}^{\infty}\frac{4^{n+1}}{(n+1)4^{n+1}}=\sum_{n=0}^{\infty}\frac{1}{n+1}$ diverges

When $x=-1$, $\displaystyle\sum_{n=0}^{\infty}\frac{(-4)^{n+1}}{(n+1)4^{n+1}}=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$ converges

Therefore,the interval of convergence is $[-1,7)$