1. (24%) Examine the series to determine whether it converges absolutely, converges conditionally, or diverges, and clearly indicate which convergence test you applied

(a) $\sum_{n=1}^{\infty} \frac{\sin[\frac{(2n-1)\pi}{2}]}{n+1}$

(b) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n}{3n+1}\right)^{2n}$

(c) $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{2\times4\times6\times\ldots\times2n}{2\times5\times8\times\ldots\times(3n-1)}$

2. (12%) Determine the interval of convergence for the power series, including testing the endpoints for convergence

(a) $\sum_{n=1}^{\infty} \frac{3^n+(-2)^n}{n}(x+1)^n$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3\times7\times11\times\ldots\times(4n-1)(x-3)^n}{3^n}$

3. (8%) Use a power series to approximate $\sin(1)$ with an error of less than 0.001

4. (15%) Evaluate the following expression. (For parts (a) and (b), you can first use the basic Taylor series to determine the original functions)

(a) $\frac{\pi}{3} - \frac{\pi^3}{3^3 3!} + \frac{\pi^5}{3^5 5!} - \frac{\pi^7}{3^7 7!} + \cdots$

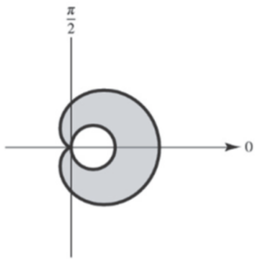
(b) $\sum_{n=1}^{\infty} \frac{2^n(n+1)}{n!}$

(c) $\lim_{x\to0} \frac{1}{\ln(1+x)} - \frac{1}{x}$

5. (12%) Derive the Maclaurin series for $f(x) = arcsin(x)$ and $g(x) = arcsin(3x^2)$. In addition, calculate $g^{(22)}(0)$ (You can use generalized binomial coefficient to represent the final results)

6. (10%) Find the arc length of the curve $x = t^2 + 1,\ y = 4t^3 + 3$ over the interval $0 \le t \le 1$

7. (10%) Find the area of the shaded region bounded by the curves $r = a(1 + cos(\theta))$ and $r = acos(\theta)$



8. (9%) Classify the following surface, if it is quadratic surface you should further classify it into six basic types of surface

(a) $x^2 + y^2 - z = 0$

(b) $r^2 = z^2 + 4$ (this representation is in cylindrical coordinates)

(c) $\rho = 4csc(\Phi)sec(\theta)$ (this representation is in spherical coordinates)

Function	Taylor series	Interval of convergence
$\frac{1}{x}$	$1-(x-1)+(x-1)^2-(x-1)^3+\cdots.+(-1)^n(x-1)^n+\cdots.$	$0<x<2$
$\frac{1}{1+x}$	$1-x+x^2-x^3+\cdots.+(-1)^n x^n+\cdots.$	$-1<x<1$
$\ln x$	$(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\cdots.+\frac{(-1)^{n-1}(x-1)^n}{n}+\cdots.$	$0<x\leq 2$
e^x	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots.+\frac{x^n}{n!}+\cdots.$	$-\infty<x<\infty$
$sin(x)$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots.+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\cdots.$	$-\infty<x<\infty$
$cos(x)$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots.+\frac{(-1)^n x^{2n}}{(2n)!}+\cdots.$	$-\infty<x<\infty$
$arctan(x)$	$x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots.+\frac{(-1)^n x^{2n+1}}{2n+1}+\cdots.$	$-1\leq \mathrm{x}\leq 1$
$arcsin(x)$	$x+\frac{x^3}{2\times 3}+\frac{1\times 3x^5}{2\times 4\times 5}+\cdots.+\frac{(2n)!\,x^{2n+1}}{(2^n n!)^2(2n+1)}+\cdots.$	$-1\leq \mathrm{x}\leq 1$
$(1+x)^k$	$1+kx+\frac{k(k-1)x^2}{2!}+\frac{k(k-1)(k-2)x^3}{3!}$ $+\cdots.+\frac{k(k-1)\ldots(k-n+1)x^n}{n!}+\cdots.$	$-1<x<1$

Derivative	Integrals
$\frac{d\sin^{-1}u}{dx}=\frac{u'}{\sqrt{1-u^2}}$	$\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\frac{u}{a}+C$
$\frac{d\,cos^{-1}u}{dx}=\frac{-u'}{\sqrt{1-u^2}}$	$\int\frac{du}{a^2+u^2}=\frac{1}{a}tan^{-1}\frac{u}{a}+C$
$\frac{d\tan^{-1}u}{dx}=\frac{u'}{1+u^2}$	$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}sec^{-1}\frac{\lvert u\rvert}{a}+C$
$\frac{d\,cot^{-1}u}{dx}=\frac{-u'}{1+u^2}$	
$\frac{d\sec^{-1}u}{dx}=\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	
$\frac{d\,csc^{-1}u}{dx}=\frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$	