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Chapter 11**VECTORS AND THE GEOMETRY OF SPACE**

11.1 Summary

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1. **Component form of a vector in the plane** If \mathbf{v} is a vector in the plane whose initial point is the origin and whose terminal point is (v_1, v_2) , then the **component form** (分量形式) of \mathbf{v} is given by

$$\mathbf{v} = \langle v_1, v_2 \rangle .$$

The coordinates v_1 and v_2 are called the **components** (分量) of \mathbf{v} . If both the initial point and the terminal point lie at the origin, then \mathbf{v} is called the **zero vector** (零向量) and is denoted by $\mathbf{0} = \langle 0, 0 \rangle$ 11

2. **Vector addition and scalar multiplication**

Let $\mathbf{u} = \langle u_1, u_2 \rangle$

and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let c be a scalar.

- (a) The **vector sum** (向量和) of \mathbf{u} and \mathbf{v} is the vector $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$.
- (b) The **scalar multiple** (純數倍) of c and \mathbf{u} is the vector $c\mathbf{u} = \langle cu_1, cu_2 \rangle$.
- (c) The **negative** (負) of \mathbf{v} is the vector $-\mathbf{v} = (-1)\mathbf{v} = \langle -v_1, -v_2 \rangle$.
- (d) The **difference** (差) of \mathbf{u} and \mathbf{v} is $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle$.

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3. **Properties of vector operations**

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be

vectors in the plane, and let c and d be scalars.

$$1. \quad \mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

Commutative Property (交換性質)

$$2. \quad (\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$$

Associative Property (結合性質)

$$3. \quad \mathbf{u} + \mathbf{0} = \mathbf{u}$$

Additive Identity Property (加法單位元)

$$4. \quad \mathbf{u} + (-\mathbf{u}) = \mathbf{0}$$

Additive Inverse Property (加法反元素)

$$5. \quad c(d\mathbf{u}) = (cd)\mathbf{u}$$

$$6. \quad (c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$$

Distributive Property (分配性質)

$$7. \quad c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$$

Distributive Property (分配性質)

$$8. \quad 1(\mathbf{u}) = \mathbf{u}, \quad 0(\mathbf{u}) = \mathbf{0}$$

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4. Length of a scalar multiple

Let \mathbf{v} be a vector and let c be a scalar.

Then

$$\|c \mathbf{v}\| = |c| \|\mathbf{v}\|. \quad |c| \text{ is the absolute value of } c.$$

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5. **Unit vector in the direction of \mathbf{v}** If \mathbf{v} is a nonzero vector in the plane, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$$

has length 1 and the same direction as \mathbf{v} 24

6. Unit vector on the unit circle: $\mathbf{u} = \langle \cos \theta, \sin \theta \rangle = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}$
 $\mathbf{v} = \|\mathbf{v}\| \langle \cos \theta, \sin \theta \rangle = \|\mathbf{v}\| \cos \theta \mathbf{i} + \|\mathbf{v}\| \sin \theta \mathbf{j}$ 30

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7. Vectors in space (空間中的向量)

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and

$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors in space and let c be a scalar.

(a) Equality of Vectors: $\mathbf{u} = \mathbf{v}$ if and only if $u_1 = v_1$, $u_2 = v_2$, and $u_3 = v_3$.

(b) Component Form: If \mathbf{v} is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, then

$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle.$$

(c) Length: $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$.

(d) Unit Vector in the Direction of \mathbf{v} : $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{1}{\|\mathbf{v}\|} \right) \langle v_1, v_2, v_3 \rangle$, $\mathbf{v} \neq \mathbf{0}$

(e) Vector Addition: $\mathbf{v} + \mathbf{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$

(f) Scalar Multiplication: $c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle$

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8. **Parallel vectors** (平行向量) Two nonzero vectors \mathbf{u} and \mathbf{v} are parallel if there is some scalar c such that $\mathbf{u} = c\mathbf{v}$ 53

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9. **Dot product** The **dot product** (内積) of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2.$$

The **dot product** (内積) of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3.$$

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10. Properties of the dot product

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors

in the plane or in space and let c be a scalar.

$$(a) \mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u} \quad \text{Commutative property}$$

$$(b) \mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} \quad \text{Distributive property}$$

$$(c) c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$$

$$(d) \mathbf{0} \cdot \mathbf{v} = 0$$

$$(e) \mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$$

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11. Angle between two vectors (兩向量間的夾角)

If θ is the angle

between two nonzero vectors \mathbf{u} and \mathbf{v} , then

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}. \quad (11.1)$$

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12. **Orthogonal vectors (正交向量)** The vectors \mathbf{u} and \mathbf{v} are **orthogonal (正交)** if $\mathbf{u} \cdot \mathbf{v} = 0$ 70
13. **Direction cosines (方向餘弦)** The angles α , β , and γ are the **direction angles (方向角)** of \mathbf{v} , and $\cos \alpha$, $\cos \beta$, and $\cos \gamma$ are the **direction cosines (方向餘弦)** of \mathbf{v} .
- $\cos \alpha = \frac{v_1}{\|\mathbf{v}\|}$ α is the angle between \mathbf{v} and \mathbf{i}
- $\cos \beta = \frac{v_2}{\|\mathbf{v}\|}$ β is the angle between \mathbf{v} and \mathbf{j}
- $\cos \gamma = \frac{v_3}{\|\mathbf{v}\|}$. γ is the angle between \mathbf{v} and \mathbf{k}
- $$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$
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14. **Projection** (投影) and **vector components** (向量分量) Let \mathbf{u} and \mathbf{v} be nonzero vectors. Moreover, let $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$, where \mathbf{w}_1 is parallel to \mathbf{v} , and \mathbf{w}_2 is orthogonal to \mathbf{v} , as shown in Figure ??.

□ \mathbf{w}_1 is called the projection of \mathbf{u} onto \mathbf{v} or the vector component of \mathbf{u} along \mathbf{v} , and is denoted by $\mathbf{w}_1 = \text{proj}_{\mathbf{v}} \mathbf{u}$.

□ $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1$ is called the vector component of \mathbf{u} orthogonal to \mathbf{v} .

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15. **Projection using the dot product** If \mathbf{u} and \mathbf{v} are nonzero vectors, then the projection of \mathbf{u} onto \mathbf{v} is given by

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v}.$$

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16. **Work (功)** The work W done by a constant force \mathbf{F} as its point of application moves along the vector \overrightarrow{PQ} is given by either of the following.

$$(a) W = \left\| \text{proj}_{\overrightarrow{PQ}} \mathbf{F} \right\| \left\| \overrightarrow{PQ} \right\| \quad (\text{Projection form})$$

$$(b) W = \mathbf{F} \cdot \overrightarrow{PQ} \quad (\text{Dot form})$$

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Section 11.4 The cross product of two vectors in space 88

17. **Cross product of two vectors in space** Let

$$\mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k} \quad \text{and} \quad \mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}$$

be vectors in space. The **cross product (外積)** of \mathbf{u} and \mathbf{v} is the vector

$$\mathbf{u} \times \mathbf{v} = (u_2 v_3 - u_3 v_2) \mathbf{i} - (u_1 v_3 - u_3 v_1) \mathbf{j} + (u_1 v_2 - u_2 v_1) \mathbf{k}.$$

18. Determinant form of $\mathbf{u} \times \mathbf{v}$

$$\begin{aligned}\mathbf{u} \times \mathbf{v} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \mathbf{k} \\ &= (u_2v_3 - u_3v_2) \mathbf{i} - (u_1v_3 - u_3v_1) \mathbf{j} + (u_1v_2 - u_2v_1) \mathbf{k}\end{aligned}$$

19. Algebraic property of the cross product (外積的代數性質)

Let \mathbf{u} , \mathbf{v} , and \mathbf{w} be vectors in space, and let c be a scalar.

(a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$

(b) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

$$(c) \ c(\mathbf{u} \times \mathbf{v}) = (c\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (c\mathbf{v})$$

$$(d) \ \mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$$

$$(e) \ \mathbf{u} \times \mathbf{u} = \mathbf{0}$$

$$(f) \ \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$$

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20. **Geometric properties of the cross product** (外積幾何性質) Let

\mathbf{u} and \mathbf{v} be nonzero vectors in space, and let θ be the angle between \mathbf{u} and \mathbf{v} .

(a) $\mathbf{u} \times \mathbf{v}$ is orthogonal to both \mathbf{u} and \mathbf{v} .

$$(b) \ \|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| \|\mathbf{v}\| \sin \theta$$

(c) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if \mathbf{u} and \mathbf{v} are scalar multiple of each other.

(d) $\|\mathbf{u} \times \mathbf{v}\| =$ area of parallelogram having \mathbf{u} and \mathbf{v} as adjacent sides.

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21. **The triple scalar product** For $\mathbf{u} = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k}$, $\mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}$, and $\mathbf{w} = w_1 \mathbf{i} + w_2 \mathbf{j} + w_3 \mathbf{k}$, the **triple scalar product** (純量三重積) is given by

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}.$$

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22. **Geometric property of triple scalar product** (三重純量積的幾何性質)

The volume V of a parallelepiped with vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} as adjacent edges is given by

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|.$$

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Section 11.5 Lines and planes in space 116

23. **Parametric equations of a line in space** A line L paralleled to the vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P(x_1, y_1, z_1)$ is represented by the **parametric equations** (參數式)

$$x = x_1 + at, \quad y = y_1 + bt, \quad \text{and} \quad z = z_1 + ct.$$

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24. **Symmetric equations** (對稱方程式) **of the line**

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad \text{Symmetric equations}$$

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25. Standard equation of a plane in space (空間中平面的標準式)

The plane containing the point (x_1, y_1, z_1) and having normal vector $\mathbf{n} = \langle a, b, c \rangle$ can be represented by the standard form of the equation of a plane

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

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26. General form (一般型) of the equation of a plane in space

$$ax + by + cz + d = 0 \quad \text{General form of equation}$$

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27. Distances between a point and a plane (平面與點的距離) The

distance between a plane and a point Q (not in the plane) is

$$D = \left\| \text{proj}_{\mathbf{n}} \overrightarrow{PQ} \right\| = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

where P is a point in the plane and \mathbf{n} is normal to the plane..... 140

28. The distance between the point $Q(x_0, y_0, z_0)$ and the plane given by $ax + by + cz + d = 0$ is

$$D = \frac{|a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1)|}{\sqrt{a^2 + b^2 + c^2}} \quad \text{or} \quad D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

where $P(x_1, y_1, z_1)$ is a point in the plane and $d = -(ax_1 + by_1 + cz_1)$.

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29. Distances between a point and a line in space (空間中線與點的距離)

The distance between a point Q and line in space is given by

$$D = \frac{\left\| \overrightarrow{PQ} \times \mathbf{u} \right\|}{\|\mathbf{u}\|}$$

where \mathbf{u} is a direction vector for the line and P is a point on the line.145

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30. **Cylinder** Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a **cylinder** (柱面). C is called the **generating curve** (母曲線) (or **directrix**) of the cylinder, and the parallel lines are called **rulings** (直紋線)..... 152
31. **Equation of cylinders** The equation of a cylinder whose ruling are parallel to one of the coordinate axes contain only the variables corre-

sponding to the other two axes. 153

32. **Quadric surface** The equation of a **quadric surface** (**二次曲面**) in space is a second-degree equation in three variables. The **general form** (**一般型**) of the equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0.$$

There are six basic types of quadric surfaces: **ellipsoid** (**橢球面**), **hyperboloid** (**單葉雙曲面**), **hyperboloid of two sheets** (**雙葉雙曲面**), **elliptic cone** (**橢錐面**), **elliptic paraboloid** (**橢圓拋物面**), and **hyperbolic paraboloid** (**雙曲拋物面**). 156

33. **Surface of revolution** If the graph of a radius function r is revolved about one of the coordinate axes, the equation of the resulting **surface of revolution** (**旋轉曲面**) has one of the following forms.

(a) Revolved about the x -axis: $y^2 + z^2 = [r(x)]^2$

(b) Revolved about the y -axis: $x^2 + z^2 = [r(y)]^2$

(c) Revolved about the z -axis: $x^2 + y^2 = [r(z)]^2$

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Section 11.7 Cylindrical and spherical coordinates 173

34. **The cylindrical coordinate system** In a cylindrical coordinate system

(圓柱座標系統), a point P in space is represented by an ordered triple (r, θ, z) .

(a) (r, θ) is a polar representation of the projection of P in the xy -plane.

(b) z is the directed distance from (r, θ) to P .

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35. Cylindrical to rectangular (圓柱到直角):

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z$$

Rectangular to cylindrical (直角到圓柱):

$$r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}, \quad z = z$$

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36. The spherical coordinate system In a spherical coordinate system (球面座標系統), a point P in space is represented by an ordered triple (ρ, θ, ϕ) .

1. ρ is the distance between P and the origin, $\rho \geq 0$.
2. θ is the same angle used in cylindrical coordinates for $r \geq 0$.
3. ϕ is the angle between the positive z -axis and the line segment \overrightarrow{OP} ,

$$0 \leq \phi \leq \pi.$$

Note that the first and third coordinates, ρ and ϕ , are nonnegative. ρ is the lowercase Greek letter rho, and ϕ is the lowercase Greek letter phi.

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37.(a) **Spherical to rectangular:**

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi.$$

(b) **Rectangular to spherical:**

$$\rho^2 = x^2 + y^2 + z^2, \quad \tan \theta = \frac{y}{x}, \quad \phi = \arccos \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right).$$

(c) **Spherical to cylindrical ($r \geq 0$):**

$$r^2 = \rho^2 \sin^2 \phi, \quad \theta = \theta, \quad z = \rho \cos \phi.$$

(d) **Cylindrical to spherical ($r \geq 0$):**

$$\rho = \sqrt{r^2 + z^2}, \quad \theta = \theta, \quad \phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right).$$

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