

## Homework6

1. Find an equation in rectangular coordinates for the surface represented by spherical equation, and sketch its graph.

$$\phi = \frac{\pi}{6}$$

2. Find  $r(t)$  that satisfies the initial conditions.

$$r'(t) = 3t^2\mathbf{j} + 6\sqrt{t}\mathbf{k}, \quad r(0) = \mathbf{i} + 2\mathbf{j}$$

3. Find the limit (if it exists).

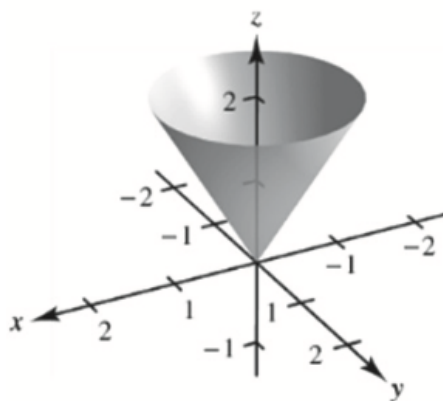
$$\lim_{t \rightarrow \infty} (e^{-t}\mathbf{i} + \frac{1}{t}\mathbf{j} + t^{\frac{1}{t}}\mathbf{k})$$

sol:

1. spherical equation

$$\begin{aligned} \cos\phi &= \frac{z}{\sqrt{x^2 + y^2 + z^2}} \\ \frac{3}{4} &= \frac{z^2}{x^2 + y^2 + z^2} \end{aligned}$$

rectangular equation,  $3x^2 + 3y^2 - z^2 = 0, z \geq 0$



2.

$$\begin{aligned} r(t) &= \int (3t^2\mathbf{j} + 6\sqrt{t}\mathbf{k}) dt = t^3\mathbf{j} + 4t^{\frac{3}{2}}\mathbf{k} + \mathbf{C} \\ r(0) &= \mathbf{C} = \mathbf{i} + 2\mathbf{j} \\ r(t) &= \mathbf{i} + (2 + t^3)\mathbf{j} + 4t^{\frac{3}{2}}\mathbf{k} \end{aligned}$$

3.

$$\lim_{t \rightarrow \infty} (e^{-t} \mathbf{i} + \frac{1}{t} \mathbf{j} + t^{\frac{1}{t}} \mathbf{k})$$

$$\text{Note that } \lim_{t \rightarrow \infty} t^{\frac{1}{t}} = 1$$

So, the limit is  $0\mathbf{i} + 0\mathbf{j} + \mathbf{k} = \mathbf{k}$