

1. (20%) Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out.)

(a)  $\lim_{(x,y) \rightarrow (0,0)} \arccos\left(\frac{x^3+y^3}{x^2+y^2}\right)$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2+y^6}$

(c)  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{e^{xyz}-1}{x^2+y^2+z^2}$

(d)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^2}$

2. (15%)

(a) Let  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{when } (x, y) \neq (0,0) \\ 0 & \text{when } (x, y) = (0,0) \end{cases}$ , evaluate  $f_x(0,0)$  and  $f_{xy}(0,0)$

- (b) Given the equation  $w - \sqrt{x-y} - \sqrt{y-z} = 0$ , differentiate implicitly to find the three first partial derivatives  $\frac{\partial w}{\partial x}$ ,  $\frac{\partial w}{\partial y}$  and  $\frac{\partial w}{\partial z}$

- (c) Find a set of parametric equations for the tangent line to the curve of intersection of the surface  $x^2 + y^2 + z^2 = 4$  and  $(x-1)^2 + y^2 = 1$  at the point  $(1, 1, \sqrt{2})$ .

3. (10%) Given  $f(x, y) = y^2 + \sin(xy)$ . Find the directions at the point  $(0,1)$  where the directional derivative of  $f(x, y)$  in that direction is 1. Express your result as unit vector.

4. (15%) Let  $f(x, y) = x^4 - 2x^2 - 2xy^2 - y^2$

(a) Find the critical points of  $f(x, y)$

- (b) Determine whether they are local maximum, local minimum or saddle points

5. (15%) Evaluate the following expression

(a)  $\int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dy dx$

(b)  $\int_1^3 \int_0^x \frac{1}{\sqrt{x^2+y^2}} dy dx$

(c)  $\int_0^{\frac{\pi}{4}} \int_0^6 \int_0^{6-r} rz dz dr d\theta$

6. (10%) Find the area of the surface given by  $z = f(x, y) = xy$  that lies above the region  $R$  where  $R = \{(x, y): x^2 + y^2 \leq 9\}$

7. (15%) Evaluate the triple integral  $\int \int \int_Q x^2 + y^2 dV$  where  $Q = \{-1 \leq x \leq 1, -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}, \sqrt{x^2+y^2} \leq z \leq 1\}$