Homework9

1. Find a set of parametric equations for the tangent line to the curve of intersection of the surfaces at the given point.

$$z = \sqrt{x^2 + y^2}, 5x - 2y + 3z = 22 \text{ , } (3,4,5)$$

2. Find relative extrema and saddle points of the function.

$$f(x, y) = -5\mathrm{x}^2 + 4\mathrm{xy} - \mathrm{y}^2 + 16\mathrm{x} + 10$$

3. Use Lagrange multipliers to find the indicated extrema of f subject to two constraints,assuming that x, y, and z are nonnegative.

Minimize : $f(x, y, z) = x^2 + y^2 + z^2$

Constraint : $x + 2z = 6, x + y = 12$

Sol:

1.

$$F(x, y, z) = \sqrt{x^2 + y^2} - z$$
$$\nabla F(x, y, z) = \frac{x}{\sqrt{x^2 + y^2}}\mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}}\mathbf{j} - \mathbf{k}$$
$$\nabla F(3,4,5) = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j} - \mathbf{k}$$
$$G(x, y, z) = 5x - 2y + 3z - 22$$
$$\nabla G(x, y, z) = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$
$$\nabla G(3,4,5) = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$
$$\nabla F \times \nabla G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{3}{5} & \frac{4}{5} & -1 \\ 5 & -2 & 3 \end{vmatrix} = \frac{2}{5}\mathbf{i} - \frac{34}{5}\mathbf{j} - \frac{26}{5}\mathbf{k}$$
$$\text{Direction numbers: } 1, -17, -13$$
$$x = 3 + t, y = 4 - 17t, z = 5 - 13t$$

2.

$$f_x = -10x + 4y + 16 = 0$$
$$f_y = 4x - 2y = 0$$

Solving simultaneously yields $x = 8, y = 16$

$f_{xx} = -10, f_{yy} = -2, f_{xy} = 4$ At the critical point $(8,16), f_{xx} < 0$ and

$f_{xx}f_{yy} - (f_{xy})^2 > 0$ So,(8,16) is a relative maximum

3.

$$
\begin{aligned}
\nabla f &= \lambda \nabla g + \mu \nabla h \\
2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k} &= \lambda(\mathbf{i} + \mathbf{2k}) + \mu(\mathbf{i} + \mathbf{j}) \\
2x &= \lambda + \mu \\
2y &= \mu \\
2z &= 2\lambda \\
2x &= 2y + z \\
x + 2z = 6 \Rightarrow z &= \frac{6-x}{2} = 3 - \frac{x}{2} \\
x + y = 12 \Rightarrow y &= 12 - x \\
2x &= 2(12 - x) + \left(\frac{x}{2}\right) \Rightarrow \frac{9}{2}x = 27 \Rightarrow x = 6 \\
\text{x} &= 6, z = 0 \\
f(6,6,0) &= 72
\end{aligned}
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