Homework8

1.Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ using the appropriate Chain Rule.

$$w = x^2 + y^2 + z^2 \quad , \; x = tsins, \; y = tcos\ s, \; z = st^2$$

2.Find the directional derivative of the function at P in the direction of $\mathbf{v}$.

$$f(x,y) = e^{-(x^2+y^2)} \; , P(0,0) \; , \mathbf{v} = \mathbf{i} + \mathbf{j}$$

3.Use the gradient to find the directional derivative of the function at P in the direction of $\overrightarrow{PQ}$

$$f(x,y,z) = ln(x+y+z), \; P(1,0,0), \; Q(4,3,1)$$

4.Find the gradient of the function and the maximum value of the directional derivative at the given point.

$$f(x,y) = \frac{x+y}{y+1}, \; (0,1)$$

sol:

1.

$$\begin{aligned}\frac{\partial w}{\partial s} &= 2x + cos\ s + 2y(-tsins) + 2z(t^2) \\ &= 2t^2\ sin\ s\ cos\ s - 2t^2\ sin\ s \cos\ s + 2st^4 \\ &= 2st^4\end{aligned}$$

$$\begin{aligned}\frac{\partial w}{\partial t} &= 2x\ sin\ s + 2y\ cos\ s + 2z(2st) \\ &= 2tsin^2\ s + 2tcos^2\ s + 4s^2t^3 \\ &= 2t + 4s^2t^3\end{aligned}$$

2.

$$\begin{aligned}\mathbf{u} &= \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} \\ D_u f(x,y) &= -2xe^{-(x^2+y^2)}\left(\frac{\sqrt{2}}{2}\right) + (-2ye^{-(x^2+y^2)})\left(\frac{\sqrt{2}}{2}\right) \\ D_u f(0,0) &= 0\end{aligned}$$

3.

$$\begin{aligned}
\mathbf{v} &= 3\mathbf{i} + 3\mathbf{j} + \mathbf{k} \\
\bigtriangledown f &= \frac{1}{x+y+z}(\mathbf{i}+\mathbf{j}+\mathbf{k}) \\
At\ (1,0,0), \bigtriangledown f &= \mathbf{i}+\mathbf{j}+\mathbf{k} \\
\mathbf{u} &= \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{1}{\sqrt{19}}(3\mathbf{i}+3\mathbf{j}+\mathbf{k}) \\
D_u f &= \bigtriangledown f \cdot \mathbf{u} = \frac{7}{\sqrt{19}}
\end{aligned}$$

4.

$$\begin{aligned}
\bigtriangledown f(x,y) &= \frac{1}{y+1}\mathbf{i} + \frac{1-x}{(y+1)^2}\mathbf{j} \\
\bigtriangledown f(0,1) &= \frac{1}{2}\mathbf{i} + \frac{1}{4}\mathbf{j} \\
||\bigtriangledown f(0,1)|| &= \sqrt{\frac{1}{4}+\frac{1}{16}} = \frac{1}{4}\sqrt{5}
\end{aligned}$$