Homework4

1. Find the nth Maclaurin polynomial for the function.

$$f(x) = \frac{1}{1-x}, n = 5$$

2. Use the power series $\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n$, $|x| < 1$ to find a power series for the function, centered at 0, and determine the interval of convergence.

$$f(x) = \ln(x^2+1)$$

3. Use the definition of Taylor series to find the Taylor series, centered at c, for the function.

$$f(x) = \ln x \ , \mathrm{c} = 1$$

Sol:

1. $f(0) = f'(0) = 1, f''(0) = 2, f'''(0) = 6, f^{(4)}(0) = 24, f^{(5)}(0) = 120$

$$P_5(x) = 1 + x + \frac{2x^2}{2!} + \frac{6x^3}{3!} + \frac{24x^4}{4!} + \frac{120x^5}{5!} = 1 + x + x^2 + x^3 + x^4 + x^5$$

2.

$$\frac{2x}{x^2+1} = 2x\sum_{n=0}^{\infty}(-1)^n x^{2n} = \sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}$$

Because $\frac{d}{dx}(\ln(x^2+1)) = \frac{2x}{x^2+1}$,you have

$$\ln(x^2+1) = \int \left[\sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}\right] dx = C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{n+1}, \quad -1 \le x \le 1$$

To solve for C ,let $x = 0$ and conclude that $C = 0$. Therefore,

$$\ln(x^2+1) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{n+1}, [-1, 1]$$

3.

For $c = 1$, you have,

$$f(1) = 0 \,, f'(1) = 1 \,, f''(1) = -1 \,, f'''(1) = 2 \,, f^{(4)}(1) = -6 \,, f^{(5)}(1) = 24$$

So,you have:

$$\ln x = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)(x-1)^n}{n!}$$

$$= 0 + (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!} - \frac{6(x-1)^4}{4!} + \frac{24(x-1)^5}{5!} - \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}$$