

Homework10_sol

1. Evaluate the iterated integral.

$$(a) \int_1^3 \int_0^y \frac{4}{x^2+y^2} dx dy$$

$$(b) \int_0^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y} dy dx$$

2. Evaluate the double integral.

$$\int_0^4 \int_0^{\sqrt{4y-y^2}} x^2 dx dy$$

Sol:

1.(a)

$$\begin{aligned} \int_1^3 \int_0^y \frac{4}{x^2+y^2} dx dy &= \int_1^3 \left[\frac{4}{y} \arctan \left(\frac{x}{y} \right) \right]_0^y dy \\ &= \int_1^3 \frac{4}{y} \left(\frac{\pi}{4} \right) dy \\ &= \int_1^3 \frac{\pi}{4} dy \\ &= [\pi \ln y]_1^3 \\ &= \pi \ln 3 \end{aligned}$$

(b)

$$\begin{aligned} \int_0^{\ln(10)} \int_{e^x}^{10} \frac{1}{\ln y} dy dx &= \int_1^{10} \int_0^{\ln y} \frac{1}{\ln y} dx dy \\ &= \int_1^{10} \left[\frac{x}{\ln y} \right]_0^{\ln y} dy \\ &= \int_1^{10} dy \\ &= [y]_1^{10} \\ &= 9 \end{aligned}$$

2.

$$\Leftrightarrow \frac{y-2}{2} = \sin \theta$$

$$\frac{1}{2} dy = \cos \theta d\theta$$

$$\begin{aligned} \int_0^4 \int_0^{\sqrt{4y-y^2}} x^2 dx dy &= \int_0^4 \frac{1}{3} (4y - y^2)^{\frac{3}{2}} dy \\ &= \frac{1}{3} \int_0^4 [4 - (y-2)^2]^{\frac{3}{2}} dy \\ &= \frac{1}{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 16 \cos^4 \theta d\theta \\ &= \frac{16}{3} \times \frac{3\pi}{8} \\ &= 2\pi \end{aligned}$$