Homework2

1. Use the Integral Test to determine the convergence or divergence of the series.

$$\frac{\ln 2}{\sqrt{2}}+\frac{\ln 3}{\sqrt{3}}+\frac{\ln 4}{\sqrt{4}}+\frac{\ln 5}{\sqrt{5}}+\frac{\ln 6}{\sqrt{6}}+\cdots$$

2. Use the Limit Comparison test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty}\frac{n}{(n+1)2^{n-1}}$$

3. Determine the convergence or divergence of the series.

(a)

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$$

(b)

$$\sum_{n=0}^{\infty}\frac{(-1)^{n}}{(2n+1)!}$$

Sol:

1.

$$\sum_{n=2}^{\infty}\frac{\ln n}{\sqrt{n}}$$

Let $f(x)=\frac{\ln x}{\sqrt{x}}, f'(x)=\frac{2-\ln x}{2x^{3/2}}$

f is positive,continuous,and decreasing for $x>e^2\approx 7.4$

$\int_8^{\infty}\frac{\ln x}{\sqrt{x}}dx=\left[2\sqrt{x}(\ln x-2)\right]_8^{\infty}=\infty$, $\sum_{n=8}^{\infty}\frac{\ln n}{\sqrt{n}}$ diverges

so $\sum_{n=2}^{\infty}\frac{\ln n}{\sqrt{n}}$ also diverges.

2.

$$\lim_{n\to\infty} \frac{n\ /\ [(n+1)2^{n-1}]}{1\ /\ (2^{n-1})} = \lim_{n\to\infty} \frac{n}{n+1} = 1$$

Therefore, $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$ converges by a limit comparison with the convergent geometric series $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$

3.

(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$$

$$\lim_{n\to\infty} \frac{n^{1/2}}{n^{1/3}} = \lim_{n\to\infty} n^{1/6} = \infty$$

Diverges by the nth-Term Test

(b)

$$a_{n+1} = \frac{1}{(2n+3)!} < \frac{1}{(2n+1)!} = a_n$$

$$\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$$

Converges by Alternating Series Test