Homework1

1. Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges,find its limit.

$$a_n = \frac{\ln(n^3)}{2n}$$

2. Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$$

Sol:

1.

$$\lim_{n\to\infty} \frac{\ln(n^3)}{2n} = \lim_{n\to\infty} \frac{3}{2}\frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{3}{2}\left(\frac{1}{n}\right) = 0 \text{ , converges}$$

2. $s_n = \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$$= 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$

$\sum_{n=1}^{\infty} \left(\frac{1}{n} + \frac{1}{n+2}\right) = \lim_{n\to\infty} s_n = \lim_{n\to\infty} 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} = \frac{3}{2}$, converges