# Contents

# List of Tables

# List of Figures

# Chapter 7

# Applications of Integration

## 7.1 Summary



1. **Area of a region between two curves** If $f$ and $g$ are continuous on $[a, b]$ and $g(x) \leq f(x)$ for all $x$ in $[a, b]$, then the area of the region bounded by the graphs of $f$ and $g$ and the vertical lines $x = a$

and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,\mathrm{d}x.$$





3. **The Disk Method (圓盤法)** To find the volume of a solid of revolution with the Disk Method, use one of the following, as shown in Figure 7.15.

| Horizontal axis of revolution | Vertical axis of revolution |
| --- | --- |
| Volume $= V = \pi \int_a^b [R(x)]^2 \,\mathrm{d}x$ | Volume $= V = \pi \int_c^d [R(y)]^2 \,\mathrm{d}y$ |

30

4. **Washer Method** (墊圈法) Consider a region bounded by an outer radius $R(x)$ and an inner radius $r(x)$. If the region is revolved about its axis of revolution, the volume of the resulting solid is given by

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) \,\mathrm{d}x.$$

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5. **Volumes of solids with known cross sections**

(a) For cross sections of area $A(x)$ taken perpendicular to the $x$-axis,

$$\text{Volume} = \int_a^b A(x)\,\mathrm{d}x.$$ **See Figure 7.24a**

(b) For cross sections of area $A(y)$ taken perpendicular to the $y$-axis,

$$\text{Volume} = \int_c^d A(y)\,\mathrm{d}y.$$ **See Figure 7.24b**



6. **The Shell Method (柱殼法)** To find the volume of a solid of revolution with the Shell Method, use one of the following, as show in

7. **Arc length** Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** (弧長) of $f$ between $a$ and $b$ is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\,\mathrm{d}x.$$

Similarly, for a smooth curve given by $x = g(y)$, the **arc length** (弧長)

of $g$ between $c$ and $d$ is

$$s = \int_c^d \sqrt{1 + [g'(y)]^2}\, \mathrm{d}y.$$



8. **Surface of revolution** If the graph of a continuous function is revolved about a line, the resulting surface is a **surface of revolution** (**旋轉面**). . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 86

9. **Area of a surface of revolution** Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area $S$ of the surface of revolution formed by revolving the graph of $f$ about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, \mathrm{d}x \qquad \text{$y$ is a function of $x$}$$

where $r(x)$ is the distance between the graph of $f$ and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1 + [g'(y)]^2}\,\mathrm{d}y$$

$x$ **is a function of** $y$

where $r(y)$ is the distance between the graph of $g$ and the axis of revolution. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 94

**Section 7.5 Work** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98

10. **Work done by a constant force** If an object is moved a distance $D$ in the direction of an applied constant force $F$, then the **work** (功) $W$ done by the force is defined as $W = FD$. . . . . . . . . . . . . . . . . . 99

11. **Work by a variable force** If an object is moved along a straight line by a continuously varying force $F(x)$, then the work $W$ done by the

force as the object is moved from $x = a$ to $x = b$ is

$$W = \lim_{\|\triangle\| \to 0} \sum_{i=1}^{n} \triangle W_i = \int_a^b F(x)\, \mathrm{d}x.$$



12. **Moments and center of mass: one-dimensional system**

Let the point masses $m_1$, $m_2$, . . . , $m_n$ be located at $x_1$, $x_2$, . . . , $x_n$.

(a) The **moment about the origin** (**對原點的力矩**) is $M_0 = m_1x_1 + m_2x_2 + \cdots + m_nx_n$.

(b) The **center of mass** (**質心**) is $\bar{x} = \dfrac{M_0}{m}$, where $m = m_1 + m_2 + \cdots + m_n$ is the **total mass** (**總質量**) of the system.

13. **Moment and center of mass: two-dimensional system**

Let the point masses $m_1$, $m_2$, . . ., $m_n$ be located at $(x_1, y_1)$, $(x_2, y_2)$, . . ., $(x_n, y_n)$.

(a) The **moment about the $y$-axis** (**對 $y$ 軸的力矩**) is $M_y = m_1x_1 + m_2x_2 + \cdots + m_nx_n$.

(b) The **moment about the $x$-axis** (**對 $x$ 軸的力矩**) is $M_x = m_1y_1 + m_2y_2 + \cdots + m_ny_n$.

(c) The **center of mass** (**重心**) $(\bar{x}, \bar{y})$ (or **center of gravity**) is

$$\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}$$

where $m = m_1 + m_2 + \cdots + m_n$ is the **total mass** (**總質量**) of the system.

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14. **Moments and center of mass of a planar lamina**

Let $f$ and $g$ be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$, and consider the planar lamina of uniform density $\rho$ bounded by the graphs of $y = f(x)$, $y = g(x)$, and $a \leq x \leq b$.

(a) The **moments about the $x$- and $y$-axes** are

$$M_x = \rho \int_a^b \left[\frac{f(x) + g(x)}{2}\right] [f(x) - g(x)] \, \mathrm{d}x$$

$$M_y = \rho \int_a^b x \, (f(x) - g(x)) \, \mathrm{d}x.$$

(b) The **center of mass** (**質心**) $(\bar{x}, \bar{y})$ is given by $\bar{x} = \dfrac{M_y}{m}$ and $\bar{y} = \dfrac{M_x}{m}$, where $m = \rho \int_a^b [f(x) - g(x)] \, \mathrm{d}x$ is the mass of the lamina.

in a liquid is

$$\text{Pressure} = P = wh$$

where $w$ is the weight-density of the liquid per unit of volume. . . . . . 154

17. **Force exerted by a fluid**

The **force $F$ exerted by a fluid** (流體施加力) of constant weight-density $w$ (per unit of volume) against a submerged vertical plane region from $y = c$ to $y = d$ is

$$F = w \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} h(y_i) L(y_i) \Delta y = w \int_c^d h(y) L(y) \, \mathrm{d}y$$

where $h(y)$ is the depth of the fluid at $y$ and $L(y)$ is the horizontal length of the region at $y$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 159

# INDEX