1

(a)

$y = 2xe^{4x}$, $y' = 2 \cdot e^{4x} + 2xe^{4x} \cdot 4 = 2e^{4x} + 8xe^{4x}$ #

(b)

$y = \log_6(x^2+5) = \frac{\ln(x^2+5)}{\ln(6)}$, $y' = \frac{1}{\ln(6)} \times \frac{1}{x^2+5} \times 2x = \frac{2x}{\ln(6)(x^2+5)}$ #

2.

(a)

令 $u = x^2$, $du = 2x\,dx$

$\int xe^{x^2}dx$

$= \int \frac{1}{2}e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$ #

(b)

$\int 2x3^{-x^2}dx = \int 2xe^{-x^2\ln 3}dx$ 令 $u = -x^2$, $du = -2x\,dx$

$= \int -e^{\ln(3)u}du = \frac{-1}{\ln(3)}e^{\ln(3)u} + C = \frac{-1}{\ln(3)}e^{-\ln(3)x^2} + C = \frac{-1}{\ln(3) \times 3^{x^2}} + C$ #

3.

(a) $\lim_{x \to 1^+} x^{\frac{1}{x-1}}$ (1^∞)

$\ln y = \ln x^{\frac{1}{x-1}} = \frac{1}{x-1}\ln x$

$\lim_{x \to 1^+} \ln y = \lim_{x \to 1^+} \frac{\ln x}{x-1}$ $\left(\frac{0}{0}\right)$ $= \lim_{x \to 1^+} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)} = \lim_{x \to 1^+} \frac{1}{x} = 1$

$\lim_{x \to 1^+} y = \lim_{x \to 1^+} e^{\ln y} = e^1$ #

(b)

$\lim_{x \to \infty} \frac{\ln x}{x^2}$ $\left(\frac{\infty}{\infty}\right)$

$= \lim_{x \to \infty} \frac{\frac{d}{dx}[\ln x]}{\frac{d}{dx}[x^2]} = \lim_{x \to \infty} \frac{1}{2x^2} = 0$ #