1.

(a) $f(x) = \ln(\sin(2x)\cos(2x))$

$$f'(x) = \frac{1}{\sin(2x)\cos(2x)} \cdot (\sin(2x)\cos(2x))'$$

$$= \frac{2\cos^2(2x) - 2\sin^2(2x)}{\sin(2x)\cos(2x)} = 2\frac{\cos(2x)}{\sin(2x)} - 2\frac{\sin(2x)}{\cos(2x)}$$

(b) $f(x) = \ln(\ln(x^2+\sin x))$

$$f'(x) = \frac{1}{\ln(x^2+\sin x)} \cdot \ln'(x^2+\sin x)$$

$$= \frac{1}{\ln(x^2+\sin x)} \cdot \frac{1}{x^2+\sin x} \cdot (x^2+\sin x)'$$

$$= \frac{1}{\ln(x^2+\sin x)} \cdot \frac{1}{x^2+\sin x} \cdot (2x+\cos x)$$

2.

(a) $$\int \frac{1}{x\ln(\frac{x}{2})}\,dx = \int \frac{1}{u}\,du = \ln|\ln(\tfrac{x}{2})| + C.$$

$u = \ln(\frac{x}{2})$

$du = \frac{\frac{1}{2}}{\frac{x}{2}}\,dx = \frac{1}{x}\,dx$

(b) $$\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{1}{1+u}\,du = -\ln|1+u| = -\ln|1+\cos x| + C$$

Let $u = \cos x$

$du = -\sin x\,dx$

3.

$$(f^{-1})'(x^5+3x+1) = \frac{1}{5x^4+3}, \quad x^5+3x+1 = 5 \Rightarrow x = 1$$

$$(f^{-1})'(5) = \frac{1}{8}$$