

(1) $f(x) = \frac{x^2}{x^2-4}$ 之圖形

① 定義域 $\{x | x \neq 2, -2\}$

② 令 $x=0 \Rightarrow y=0$

③ $\because \lim_{x \rightarrow \infty} f(x) = 1 \Rightarrow y=1$ 為水平漸近線

$\because \lim_{x \rightarrow 2^+} f(x) = \infty \Rightarrow x=2$ 為鉛直漸近線

$\lim_{x \rightarrow -2^+} f(x) = -\infty \Rightarrow x=-2$ 為鉛直漸近線

④ $f'(x) = \frac{-8x}{(x^2-4)^2}$

$f'(x) = 0 \Rightarrow x=0$

$f'(x)$ 不存在 $\Rightarrow x=2, -2$

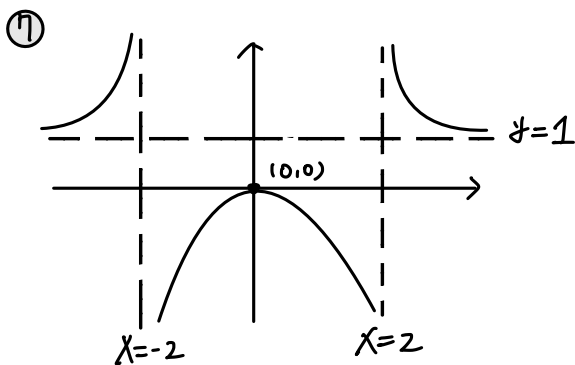
⑤ $f''(x) = \frac{24x^2+32}{(x^2-4)^3}$

$f''(x)$ 不存在 $\Rightarrow x=2, -2$

根據①~⑤建表。

⑥

x		-2		0		2	
f'	+	X	+	0	-	X	-
f''	+	X	-	-	-	X	+
f	↖	X	↘	0	↗	X	↖
	↖		↘		↗		↖
	increasing		decreasing		increasing		decreasing



(2) 求 $\sqrt[4]{625}$ 的近似值.

$$\text{令 } f(x) = \sqrt[4]{x} \Rightarrow f'(x) = \frac{1}{4} \frac{1}{\sqrt[4]{x^3}}$$

$$\therefore f(c+\Delta x) \approx f(c) + f'(c)\Delta x \Rightarrow \sqrt[4]{c+\Delta x} \approx \sqrt[4]{c} + \frac{\Delta x}{4\sqrt[4]{c^3}}$$

$$\text{取 } c=625, \Delta x=-4, \text{ 得 } \sqrt[4]{625} + \frac{-4}{4\sqrt[4]{625^3}} = 5 - \frac{1}{125} \approx 4.992$$

(3) 令 $X = \sqrt[3]{28}$, 则 $f(x) = x^3 - 28$, $f'(x) = 3x^2$

$$\text{取 } X_1 = 3, \text{ 则 } X_2 = X_1 - \frac{f(X_1)}{f'(X_1)} = 3 - \frac{-1}{3 \cdot 3^2} = 3.04$$

$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)} = 3.04 - \frac{(3.04)^3 - 28}{3 \cdot (3.04)^2} = 3.04 - \frac{0.094}{27.725}$$

$$\approx 3.0366.$$