1. (a) $f(x)$ is continuous on $[1,2]$ and is differentiable on $(1,2)$ so the Mean Value Theorem is applicable.

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{2-0}{2-1} = 2$$

$$f'(x) = 2x^3-5x+2 \Rightarrow 2x^3-5x+2=2$$

$$\Rightarrow 2x^3-5x=0 \Rightarrow x = \pm\sqrt{\tfrac{5}{2}} \text{ or } 0$$

Only $\sqrt{\frac{5}{2}}$ in the interval $[1,2]$

$\therefore c=\sqrt{\frac{5}{2}}$ and $f'(\sqrt{\frac{5}{2}})=2$ #

(b) $f(x)$ is continuous on $[2,6]$ and is differentiable on $(2,6)$ so the Mean Value Theorem is applicable.

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{-7-(-7)}{4} = 0$$

$$f'(x) = 2x-8 \Rightarrow 2x-8=0$$

$$\Rightarrow 2x=8 \Rightarrow x=4$$

Ans $c=4$ and $f'(4)=0$ #

2. x^3+3x^2+5

(1)

$$f'(x) = 3x^2+6x$$

令 $3x^2+6x=0 \Rightarrow x(3x+6)=0$

$x=0$ or -2

Interval	$(-\infty,-2)$	$[-2,0]$	$(0,\infty)$
$f'(x)$	+	−	+
	increasing	decreasing	increasing

Ans: Increasing on $(-\infty,-2)$ and $(0,\infty)$

Decreasing on $[-2,0]$

(2)

$$f''(x) = 6x+6$$

令 $6x+6=0 \Rightarrow x=-1$

inflection point $(-1,7)$

Interval	$(-\infty,-1)$	$(-1,\infty)$
$f''(x)$	−	+
	downward	upward

Ans: Concave upward on $(-1,\infty)$

Concave downward on $(-\infty,-1)$

(3)

point	$(0,5)$	$(-2,9)$
$f''(x)$	+	−
	minimum	maximum

Ans: relative maximum $(-2,9)$

relative minimum $(0,5)$

3 (a)

$$\lim_{x \to \infty} \frac{-4x^2+2x-5}{x} = \lim_{x \to \infty} \frac{-4x+2-\frac{5}{x}}{1} = -\infty \#$$

(b) $$\lim_{x \to \infty} \frac{-4x^2-2x-5}{x^2} = \lim_{x \to \infty} \frac{-4-\frac{2}{x}-\frac{5}{x^2}}{1} = -4 \#$$

(c) $$\lim_{x \to \infty} \frac{-4x^2-2x-5}{x^3} = \lim_{x \to \infty} \frac{\frac{-4}{x}-\frac{2}{x^2}-\frac{5}{x^3}}{1} = 0 \#$$