1.

$$f(x) = \begin{cases} -2, & \text{if } x \le -1 \\ ax-b, & \text{if } -1 < x < 1 \\ 3, & \text{if } x \ge 1 \end{cases}$$

$\because$ f(x) is conti at every x

$$\therefore \lim_{x \to -1^-} -2 = \lim_{x \to -1^+} ax-b$$

$$\& \lim_{x \to 1^+} 3 = \lim_{x \to 1^-} ax-b$$

$$\therefore \begin{cases} -2 = -a-b \\ 3 = a-b \end{cases}$$

$$\therefore \begin{cases} a = \frac{5}{2} \\ b = -\frac{1}{2} \end{cases}$$

2.

$f(x) = x^2 - 2x - 3$

$f(0) = -3$, $f(4) = 5$

$\because f(x)$ is conti. in $[0,4]$ & $f(0) < f(c) < f(4)$

$\therefore$ by IVT , $\exists\, c \in [0,4]$ s.t. $f(c) = 0$

$f(c) = c^2 - 2c - 3 = 0$

$\Rightarrow (c-3)(c+1) = 0 \Rightarrow c = 3$ or $-1.$ (X)

Hence $c = 3$ #

3.

(a) $\lim\limits_{a \to 0^+} \frac{1}{3x} = \infty$

(b) $\lim\limits_{x \to -5^-} \frac{3x}{2x+10} = \infty .$