a.

$u = \ln\sqrt{x}$ $\quad du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}x^{-1}$

$dv = \sqrt{x}$ $\quad v = \frac{2}{3}x^{\frac{3}{2}}$

$$\int \sqrt{x}\ln\sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\ln\sqrt{x} - \frac{1}{3}\int x^{\frac{1}{2}}dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln\sqrt{x} - \frac{2}{9}x^{\frac{3}{2}} + C$$

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b. $\int \frac{1}{x^2-4x+9}\,dx$

$$= \int \frac{1}{(x^2-4x+4)+9-4}\,dx = \int \frac{1}{(x-2)^2+5}\,dx = \frac{1}{\sqrt{5}}\int \frac{1}{\sqrt{5}}\frac{1}{(\frac{x-2}{\sqrt{5}})^2+1}\,dx$$

$$= \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-2}{\sqrt{5}}\right) + C.$$

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c. $\int x \cdot e^x \sin x\,dx$

let $u = xe^x$ $\quad du = e^x + xe^x$

$dv = \sin x$ $\quad v = -\cos x$

$$= -xe^{-x}\cdot\cos x - \int e^x + xe^x \cdot (-\cos x)\,dx$$

$$= -xe^{-x}\cos x - \int e^x dx + \int xe^x (\cos x)\,dx$$

$u = xe^x$ $\quad du = e^x + xe^x$

$dv = \cos x$ $\quad v = \sin x$

$$= -xe^{-x}\cos x - \int e^x dx + \int xe^x \cos x\,dx$$

$$= -xe^{-x}\cos x - \int e^x dx - xe^x \sin x - \int e^x + xe^x \sin x\,dx$$

$$= -xe^{-x}(\cos x + \sin x) - 2\int e^x dx - \int xe^x \sin x\,dx$$

$$\Rightarrow \int xe^x \sin x\,dx = \frac{1}{2}\left(-xe^{-x}(\cos x + \sin x) - 2e^x + C\right)$$

d.

$$\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx$$

Let $u=\cos x$

$-du = \sin x\,dx$

$$= -\int (1-u^2)\,du$$

$$= -u + \frac{1}{3}u^3 + C$$

$$= \boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$$

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2. $$\int \sec^n(x)\,dx = \int \sec^{(n-2)}x \sec^2 x\,dx$$

$u = \sec^{(n-2)}x$

$du = (n-2)\sec^{n-2}x \tan^{n-2} x$

$dv = \sec^2 x$, $v = \tan x$

$$= \sec^{(n-2)}x\tan x - \int (n-2)\sec^{n-2}x\tan x\,dx$$

$$= \sec^{(n-2)}x\tan x - (n-2)\int \sec^{(n-2)}x(\sec^2 x - 1)\,dx$$

$$= \sec^{(n-2)}x\tan x - (n-2)\int \sec^n x + \sec^{n-2}x\,dx$$

$$= \sec^{(n-2)}x\tan x - (n-2)\int \sec^n x\,dx + (n-2)\int \sec^{n-2}x\,dx$$

$$(n-1)I_n = \sec^{(n-2)}x\tan x + (n-2)I_{n-2}$$

$$I_n = \boxed{\frac{1}{n-1}\sec^{(n-2)}x\tan x + \frac{n-2}{n-1}I_{n-2}}$$