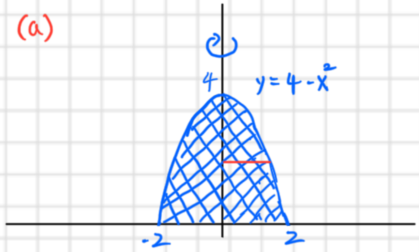
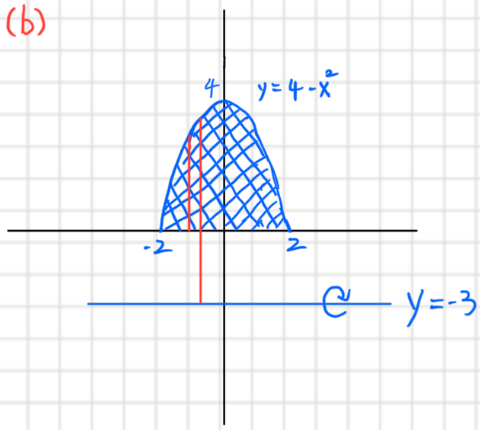
1.

(a) $x=\sqrt{4-y}$

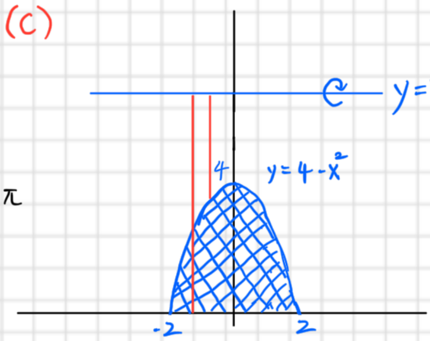
$$\int_0^4 \pi(\sqrt{4-y})^2\,dy = \pi\int_0^4 (4-y)\,dy = \pi\left[4y-\frac{y^2}{2}\right]_0^4 = \pi\left[(16-8)-0\right] = 8\pi \;\#$$



(b)
$$\int_{-2}^{2} \pi\left[(4-x^2)+3\right]^2 dx - \int_{-2}^{2}\pi(3^2)\,dx = \pi\int_{-2}^{2} 49-14x^2+x^4\,dx - \pi\int_{-2}^{2} 9\,dx$$
$$=\pi\left[49x-\frac{14}{3}x^3+\frac{1}{5}x^5\right]_{-2}^{2} - \pi\left[9x\right]_{-2}^{2}$$
$$=\pi\left[\left(98-\frac{112}{3}+\frac{32}{5}\right)-\left(-98+\frac{112}{3}-\frac{32}{5}\right)\right]-36\pi$$
$$=\pi\left[\left(196-\frac{224}{3}+\frac{64}{5}\right)\right]-36\pi = \frac{2012}{15}\pi-36\pi = \frac{1472}{15}\pi \;\#$$



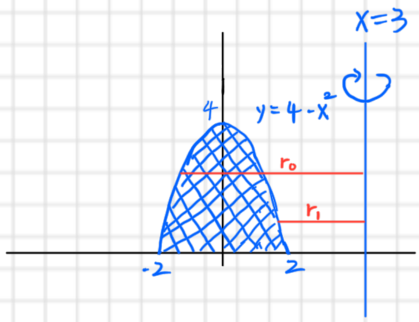
(c)
$$\int_{-2}^{2}\pi(7)^2dx - \int_{-2}^{2}\pi(7-(4-x^2))^2dx = \pi\int_{-2}^{2}49\,dx-\pi\int_{-2}^{2}(3+x^2)^2dx = \pi\int_{-2}^{2}49\,dx-\pi\int_{-2}^{2}9+6x^2+x^4\,dx$$
$$=\pi\left[49x\right]_{-2}^{2}-\pi\left[9x+2x^3+\frac{1}{5}x^5\right]_{-2}^{2} = 196\pi-\left[\left(18+16+\frac{32}{5}\right)-\left(-18-16-\frac{32}{5}\right)\right]\pi$$
$$=196\pi-\frac{404}{5}\pi = \frac{576}{5}\pi \;\#$$



(d) $r_o = 3-(-\sqrt{4-y}) = 3+\sqrt{4-y}$

$r_1 = 3-\sqrt{4-y}$

$$\int_0^4\pi(3+\sqrt{4-y})^2dy - \int_0^4\pi(3-\sqrt{4-y})^2dy$$
$$=\pi\int_0^4 13+6\sqrt{4-y}-y\,dy-\pi\int_0^4 13-6\sqrt{4-y}-y\,dy$$
$$=\pi\left[13y-4\sqrt{4-y}(4-y)-\frac{y^2}{2}\right]_0^4-\pi\left[13y+4\sqrt{4-y}(4-y)-\frac{y^2}{2}\right]_0^4 = \left[(44-(-32))-(44-32)\right]\pi = 64\pi$$



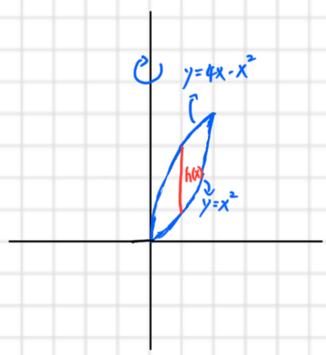
2.

(a)

$h(x) = 4x-x^2-x^2 = 4x-2x^2$

$p(x) = x$

$$2\pi\int_0^2 x(4x-2x^2)\,dx = 2\pi\int_0^2 4x^2-2x^3\,dx = 2\pi\left[\frac{4}{3}x^3-\frac{1}{2}x^4\right]_0^2$$
$$=2\pi\left[\left(\frac{32}{3}-8\right)-0\right] = \frac{16}{3}\pi$$

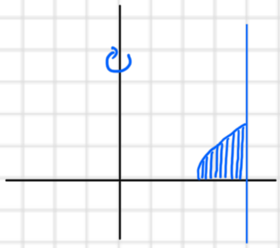


(b)

$$2\pi\int_{2.5}^{4} x\sqrt{2x-5}\,dx \qquad u=2x-5 \quad du=2\,dx \qquad x=\frac{u+5}{2}$$

$$=2\pi\int_0^3 \frac{u+5}{4}\sqrt{u}\,du = 2\pi\int_0^3 \frac{u\sqrt{u}+5\sqrt{u}}{4}\,du = 2\pi\int_0^3 \frac{u^{\frac{3}{2}}}{4}+\frac{5u^{\frac{1}{2}}}{4}\,du$$

$$=2\pi\left[\frac{u^{\frac{5}{2}}}{10}+\frac{5u^{\frac{3}{2}}}{6}\right]_0^3 = 2\pi\left[\frac{\sqrt{243}}{10}+\frac{5\sqrt{27}}{6}\right] = 2\pi\left[\frac{9\sqrt{3}}{10}+\frac{5\sqrt{3}}{2}\right] = \frac{34\sqrt{3}}{5}\pi \ \#$$



3

(a)

$$\frac{dy}{dx} = x^{-\frac{1}{3}}$$

$$\int_1^{27}\sqrt{1+(x^{-\frac{1}{3}})^2}\,dx = \int_1^{27}\sqrt{1+x^{-\frac{2}{3}}}\,dx = \int_1^{27}\sqrt{\frac{x^{\frac{2}{3}}+1}{x^{\frac{2}{3}}}}\,dx, \qquad 令\ u=x^{\frac{2}{3}}+1,\ du=\frac{2}{3}x^{-\frac{1}{3}}\,dx$$

$$=\int_2^{10}\frac{3u^{\frac{1}{2}}}{2}\,du = \left[u^{\frac{3}{2}}\right]_2^{10} = 10\sqrt{10}-2\sqrt{2} \ \#$$

(b)

$$\frac{dx}{dy} = \frac{1}{2}(y^2+2)^{\frac{1}{2}}\cdot 2y = y\sqrt{y^2+2}$$

$$\int_0^4\sqrt{1+(y\sqrt{y^2+2})^2}\,dy = \int_0^4\sqrt{1+y^4+2y^2}\,dy \qquad ※\ (y^2+1)^2 = y^4+2y^2+1$$

$$=\int_0^4 y^2+1\,dy = \left[\frac{y^3}{3}+y\right]_0^4 = \frac{76}{3} \ \#$$