1.

a). $f(x) = x \sin^{-1}x + \sqrt{1-x^2}$

$= \sin^{-1}x + x \cdot \dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{2}\dfrac{-2x}{\sqrt{1-x^2}}$

$= \sin^{-1}x + x \cdot \dfrac{1}{\sqrt{1-x^2}} - \dfrac{x}{\sqrt{1-x^2}} = \boxed{\sin^{-1}x}$

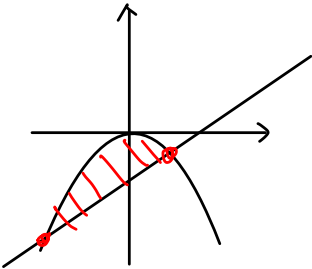b) $f(x) = \tan^{-1}x + \tan^{-1}\dfrac{1}{x}$

$= \dfrac{1}{1+x^2} + \dfrac{1}{1+(\frac{1}{x})^2} \cdot -x^{-2} = \boxed{0}$

2.

a) $\displaystyle\int \dfrac{x}{1+x^4}\,dx = \dfrac{1}{2}\int \dfrac{1}{1+u^2}\,du = \boxed{\dfrac{1}{2}\tan^{-1}x^2 + C}$ let $u = x^2$
         $du = 2x\,dx$

b) $\displaystyle\int \dfrac{1}{\sqrt{4x-x^2}}\,dx = \int \dfrac{1}{\sqrt{1-(\frac{x-2}{2})^2}}\,\dfrac{1}{2}\,dx = \boxed{\sin^{-1}(\dfrac{x-2}{2}) + C}$

3.

$(-x^2) - (x-6) = -x^2 - x + 6 = -(x+3)(x-2) \Rightarrow x = -3 \text{ or } 2$

$\displaystyle\int_{-3}^{2} -x^2 - x + 6 \, dx$

$= -\dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 + 6x \Big|_{-3}^{2}$

$= 19 + \dfrac{11}{6} = \boxed{\dfrac{125}{6}}$