

1. (20%) Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out).

(a) $\lim_{x \rightarrow 0^+} \frac{\int_0^{x^2} (\sin(\sqrt{t}) - \sqrt{t}) dt}{\int_0^{x^2} (\tan(\sqrt{t}) - \sqrt{t}) dt}$

(b) $\lim_{x \rightarrow 0^+} \tan(x) \ln(\sin^2(x))$

(c) $\lim_{x \rightarrow 1^+} (x - 1)^{\ln x}$

(d) $\lim_{x \rightarrow \infty} (e^x - x^2)$

2. (10%) Solve the following problems:

(a) Show that $f(x) = \int_1^x \sqrt{1+t^2} dt$ has an inverse function

(b) Find $(f^{-1})'(0)$

3. (15%) Evaluate the following integrals. (Hint: Try to use change of variables for all the problems)

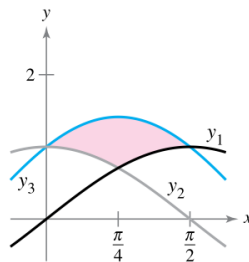
(a) $\int_3^4 (4-x)6^{(4-x)^2} dx$

(b) $\int \sqrt{e^t - 3} dt$

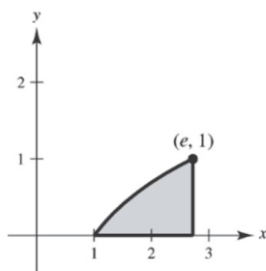
(c) $\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}}$

4. (7%) Find the area of the given region bounded by the graph y_1, y_2 and y_3

$$y_1 = \sin(x), y_2 = \cos(x), y_3 = \cos(x) + \sin(x)$$



5. (8%) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \ln x$, $y = 0$ and $x = e$ about the x -axis.



6. (6%) Use the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the y -axis.

$$y = \frac{1}{x^2}, y = 0, x = 2, x = 5$$

7. (9%) Find the arc length of the graph of the function $y = \ln(1 - x^2)$ on the interval $0 \leq x \leq \frac{1}{3}$.

8. (25%) Evaluate the following integrals. (25%)

(a) $\int \frac{\ln x}{x^3} dx$

(b) $\int \sin^2(x) \cos^3(x) dx$

(c) $\int \frac{1}{\sqrt{1+\sqrt{x}}} dx$

(d) $\int \frac{1}{1+\tan(\theta)} d\theta$

(e) $\int_1^4 \frac{1}{(x-2)^2} dx$

Derivative	Integrals
$\frac{d \sin^{-1} u}{dx} = \frac{u'}{\sqrt{1-u^2}}$	$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + C$
$\frac{d \cos^{-1} u}{dx} = \frac{-u'}{\sqrt{1-u^2}}$	$\int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
$\frac{d \tan^{-1} u}{dx} = \frac{u'}{1+u^2}$	$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{ u }{a} + C$
$\frac{d \cot^{-1} u}{dx} = \frac{-u'}{1+u^2}$	
$\frac{d \sec^{-1} u}{dx} = \frac{u'}{ u \sqrt{u^2-1}}$	
$\frac{d \csc^{-1} u}{dx} = \frac{-u'}{ u \sqrt{u^2-1}}$	