If the limit does not exist or has an infinite limit, you should point it out. In addition, do not use the L'Hôpital's rule to solve the limit problem.

1. (16%) Find the following limit

(a) $\lim_{x\to 2} \frac{2x^3-3x^2-3x+2}{x^2+x-6}$

(b) $\lim_{x\to 0} \frac{(\sqrt{16+x}-4)}{x}$

(c) $\lim_{x\to \infty} \sqrt{3x^2+1}\tan\frac{1}{x}$

(d) $\lim_{x\to 0} x\sqrt{1+\frac{4}{x^2}}$

2. (10%) Assume the following function is a differentiable function

$$f(x)=\begin{cases} x^2 sin(\frac{1}{x}), & x>0 \\ ax+b, & x\le 0 \end{cases}$$

What is the value of a and b ?

3. (9%) Assume $f(1)=8$ and $\forall x \in (1,4)$ we have $f'(x)\ge 2$. What is the minimum possible value for $f(4)$ (Hint: use the mean value theorem)

4. (12%) Remember that you can solve the derivative using the definition or the differentiation rule for the following question.

(e) Let $f(x)=\frac{x(x-2)(x-3)(x-4)}{(x+2)(x+3)(x+4)}$, find $f'(2)$.

(f) Find the derivative of $f(x)=2csc^2(\pi x)$

(g) Let $x^3+y^3=2$, find the value of $\frac{d^2y}{dx^2}$ when $x=1$

5. (20%) Let $f(x) = x^2 + \frac{1}{x}$

(a) Find the critical numbers and the possible points of inflection of $f(x)$
(b) Find the open intervals on which f is increasing or decreasing
(c) Find the open intervals of concavity
(d) Find all the asymptotes (Vertical/horizontal/Slant)
(e) Sketch the graph of $f(x)$ (Label any intercepts, relative extrema, points of inflection,and asymptotes)

6. (9%) Find a point on the graph $x = \sqrt{10y}$ that is closetst to point (0,4). (Becarefull about the domain of the function)

7. (9%) Use differential to approximat $tan(46°)$

8. (15%) Remember the meaning and the definition of definite integral when solving the following question

(a) $\int 3 + cot^2(t)dt$

(b) $\int_0^5 5 - |x - 5|dx$

(c) $\lim_{n\to\infty} 2(\frac{1+2+\cdots+n}{n^2})$