1. Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out. In addition, also remember the definition of definite integral). (20%)

(a) $\lim_{n\to\infty} \frac{1}{\sqrt{2n\cdot 1-1^2}}+\frac{1}{\sqrt{2n\cdot 2-2^2}}+\cdots+\frac{1}{\sqrt{2n\cdot n-n^2}}$

(b) $\lim_{x\to a} \frac{x\int_a^x f(t)dt}{x-a}$

(c) $\lim_{x\to 0^+} \frac{\ln(sinx)}{\ln(tanx)}$

(d) $\lim_{x\to\infty} (1+\frac{3}{x}+\frac{5}{x^2})^x$

(e) $\lim_{x\to 0} \left(\frac{1}{2x}-\frac{1}{1-e^{-2x}}\right)$

2. Let $f(x) = 3 + x + e^x$ (9%)
   (a) What is the value of $f^{-1}(x)$ when $x = 4$
   (b) What is the value of $(f^{-1})'(x)$ when $x = 4$
   (c) What is the value of $(f^{-1})''(x)$ when $x = 4$

3. Use the Mean Value Theorem to prove that $\forall a \geq 0$, we have $\frac{a}{1+a^2} \leq \tan^{-1} a \leq a$. (Hunt: use the theorem in the interval $(0, a)$) (8%)

4. Evaluate the following integral. (Hint: Try to use change of variables for all the problems) (15%)
   (a) $\int x \cdot 10^{x^2} dx$
   (b) $\int \sqrt{1+e^{2x}} dx$
   (c) $\int \frac{\sin(x)\cos(x)}{1+sin^4(x)} dx$

5. Find the equation of the tangent line $tan^{-1}(xy) = sin^{-1}(x+y)$ at $(0,0)$. (8%)

6. Evaluate the following integral. (16%)

(a) $\int \frac{\ln x}{x^2} dx$

(b) $\int_0^1 \ln(x^2+1)\, dx$

(c) $\int_0^{\frac{\pi}{4}} tan^3\theta sec^2\theta d\theta$

(d) $\int \frac{1}{x^2\sqrt{x^2+4}} dx$

7. Let $f(x) = \frac{-8x^2-7x+3}{(x+1)(x+2)(x^2+1)}$. (9%)

(a) Solve $\int f(x) dx$

(b) Solve $\int_0^{\infty} f(x) dx$

8. Determine whether the following integral diverges or converges. (9%)

(a) $\int_1^9 \frac{1}{\sqrt[3]{x-1}} dx$

(b) $\int_1^{\infty} \frac{1}{1+e^x} dx$

(c) $\int_1^{\infty} \frac{\sqrt{1+\frac{1}{x^4}}}{x} dx$

9. Find the volume of the solid generated by revolving the region bounded by the graphs of $y \le xe^{-x}, y \ge 0$ and $x \ge 0$ about the x-axis. (6%)

Derivative	Integrals
$\frac{d\sin^{-1}u}{dx}=\frac{u'}{\sqrt{1-u^2}}$	$\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\frac{u}{a}+C$
$\frac{d\,cos^{-1}u}{dx}=\frac{-u'}{\sqrt{1-u^2}}$	$\int\frac{du}{a^2+u^2}=\frac{1}{a}tan^{-1}\frac{u}{a}+C$
$\frac{d\tan^{-1}u}{dx}=\frac{u'}{1+u^2}$	$\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}sec^{-1}\frac{\lvert u\rvert}{a}+C$
$\frac{d\,cot^{-1}u}{dx}=\frac{-u'}{1+u^2}$	
$\frac{d\sec^{-1}u}{dx}=\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	
$\frac{d\,csc^{-1}u}{dx}=\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	