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Chapter 8

Integration Techniques and Improper Integrals

8.1 Summary

Section 8.1 Basic integration rules 2

1. **Review of basic integration rules ($a > 0$)**

1. $\int kf(u)\,\mathrm{d}u = k\int f(u)\,\mathrm{d}u$	2. $\int [f(u) \pm g(u)]\,\mathrm{d}u = \int f(u)\,\mathrm{d}u$
3. $\int \mathrm{d}u = u + C$	4. $\int u^n\,\mathrm{d}u = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$
5. $\int \frac{\mathrm{d}u}{u} = \ln\lvert u\rvert + C$	6. $\int e^u\,\mathrm{d}u = e^u + C$
7. $\int a^u\,\mathrm{d}u = \left(\frac{1}{\ln a}\right) a^u + C$	8. $\int \sin u\,\mathrm{d}u = -\cos u + C$
9. $\int \cos u\,\mathrm{d}u = \sin u + C$	10. $\int \tan u\,\mathrm{d}u = -\ln\lvert\cos u\rvert + C$
11. $\int \cot u\,\mathrm{d}u = \ln\lvert\sin u\rvert + C$	12. $\int \sec u\,\mathrm{d}u = \ln\lvert\sec u + \tan u\rvert$
13. $\int \csc u\,\mathrm{d}u = -\ln\lvert\csc u + \cot u\rvert + C$	14. $\int \sec^2 u\,\mathrm{d}u = \tan u + C$
15. $\int \csc^2 u\,\mathrm{d}u = -\cot u + C$	16. $\int \sec u \tan u\,\mathrm{d}u = \sec u + C$
17. $\int \csc u \cot u\,\mathrm{d}u = -\csc u + C$	18. $\int \frac{\mathrm{d}u}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$
19. $\int \frac{\mathrm{d}u}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$	20. $\int \frac{\mathrm{d}u}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{\lvert u\rvert}{a} + C$

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2. Procedures for fitting integrands to basic integration

Technique	Example	
Expand (numerator).	$(1+e^x)^2 = 1 + 2e^x + e^{2x}$	
Separate numerator.	$\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$	
Complete the square.	$\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$	
Divide improper rational function.	$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$	
Add and subtract terms in numerator.	$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$	12
Use trigonometric identities.	$\cot^2 x = \csc^2 x - 1$	
Multiply and divide by Pythagorean conjugate	$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$	

Section 8.2 Integration by parts 12

3. **Integration by parts (分部積分)** If u and v are functions of x and have continuous derivatives, then

$$\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u.$$

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4. **Guidelines for integration by parts**

   (a) Try letting $\mathrm{d}v$ be the most complicated portion of the integration rule. Then u will be remaining factor(s) of the integrand.

   (b) Trying letting u be the portion of the integrated whose derivative is a function simpler than u. Then $\mathrm{d}v$ will be the remaining factor(s) of the integrand.

   Note that $\mathrm{d}v$ always includes the $\mathrm{d}x$ of the original integrand........15

5. **A perspective: On integrating an inverse function** Let the function be strictly increasing and differentiable; the case of f strictly decreasing is similar. The area of the region marked P is the area under the curve $x = f^{-1}(y)$ from $y = a$ to $y = b$. That is, we compute the area by interchanging the roles of x and y in the usual computation of area under a curve. Thus

$$\text{area of } P = \int_a^b f^{-1}(y)\,\mathrm{d}y.$$

The area of Q is computed in the usual way:

$$\text{area of } Q = \int_{f^{-1}(a)}^{f^{-1}(b)} f(x)\,\mathrm{d}x.$$

Finally, the region marked R is a rectangle, so

$$\text{area of } R = \text{base} \times \text{height} \ = f^{-1}(a) \times a = af^{-1}(a).$$

Now, the region $P + Q + R$ is a larger rectangle with base $f^{-1}(b)$ and height b. Thus,

$$\text{area of } P = \int_a^b f^{-1}(y)\,\mathrm{d}y = bf^{-1}(b) - af^{-1}(a) - \int_{f^{-1}(a)}^{f^{-1}(b)} f(x)\,\mathrm{d}x.$$

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6. Summary of common integrals using integration by parts

(a) For integrals of the form

$$\int x^n e^{ax}\,\mathrm{d}x, \quad \int x^n \sin ax\,\mathrm{d}x, \quad \text{or} \quad \int x^n \cos ax\,\mathrm{d}x$$

let $u = x^n$ and let $\mathrm{d}v = e^{ax}\,\mathrm{d}x$, $\sin ax\,\mathrm{d}x$, $\cos ax\,\mathrm{d}x$.

(b) For integrals of the form

$$\int x^n \ln x\,\mathrm{d}x, \quad \int x^n \arcsin ax\,\mathrm{d}x, \quad \text{or} \quad \int x^n \arctan ax\,\mathrm{d}x$$

let $u = \ln x$, $\arcsin ax$, or $\arctan x$ and let $\mathrm{d}v = x^n\,\mathrm{d}x$.

(c) For integrals of the form

$$\int e^{ax}\sin bx\,\mathrm{d}x, \quad \text{or} \quad \int e^{ax}\cos bx\,\mathrm{d}x$$

let $u = \sin bx$ or $\cos bx$ and let $\mathrm{d}v = e^{ax}\,\mathrm{d}x$.

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Section 8.3 Trigonometric integrals 33

7. To break up $\int \sin^m x \cos^n x\,\mathrm{d}x$ into forms to which you can apply the

Power Rule, use the following identities.

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

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8. **Guidelines for evaluating integrals involving powers of sine and cos**

(a) If the power of the sine is odd and positive, save one sine factor and convert the remaining factors to cosines. Then, expand and integrate.

$$\int \sin^{\overbrace{2k+1}^{\text{Odd}}} x \cos^n x \, dx = \int \overbrace{(\sin^2 x)^k}^{\text{Convert to cosines}} \cos^n x \overbrace{\sin x \, dx}^{\text{Save for } du}$$

$$= \int (1 - \cos^2 x)^k \cos^n x \sin x \, dx$$

(b) If the power of the cosine is odd and positive, save one cosine factor and convert the remaining factors to sines. Then, expand and integrate.

$$\begin{aligned}\int \sin^m x \cos^{\overbrace{2k+1}^{\text{Odd}}} x \, \mathrm{d}x &= \int \sin^m x \quad \overbrace{(\cos^2 x)^k}^{\text{Convert to cosines}} \quad \overbrace{\cos x \, \mathrm{d}x}^{\text{Save for } \mathrm{d}u} \\ &= \int \sin^m x (1 - \sin^2 x)^k \cos x \, \mathrm{d}x\end{aligned}$$

(c) If the power of both the sine and cosine are even and nonnegative, make repeated use of the identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \text{and} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

to convert the integrand to odd powers of the cosine. Then proceed as in guideline 2.

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9. **Wallis's Formulas** (**華利斯公式**) If n is odd ($n \geq 3$), then

$$\int_0^{\pi/2} \cos^n x \, \mathrm{d}x = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right).$$

If n is even ($n \geq 2$), then

$$\int_0^{\pi/2} \cos^n x \, \mathrm{d}x = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right).$$

These formulas are also valid if $\cos^n x$ is replaced by $\sin^n x$. 42

10. **Guidelines for evaluating integrals involving powers of secant and t**

(a) If the power of the secant is even and positive, save a secant-squared factor and convert the remaining factors to tangents. Then, expand

and integrate.

$$\int \sec^{\overbrace{2k}^{\text{even}}} x \tan^n x \, dx = \int \overbrace{(\sec^2 x)^{k-1}}^{\text{Convert to tangents}} \tan^n x \overbrace{\sec^2 x \, dx}^{\text{Save for } du}$$
$$= \int (1 + \tan^2 x)^{k-1} \tan^n x \sec^2 x \, dx$$

(b) If the power of the tangent is odd and positive, save a secant-tangent factor and convert the remaining factors to secants. Then, expand and integrate.

$$\int \sec^m x \tan^{\overbrace{2k+1}^{\text{Odd}}} x \, dx = \int \sec^{m-1} x \overbrace{(\tan^2 x)^k}^{\text{Convert to secants}} \overbrace{\sec x \tan x \, dx}^{\text{Save for } du}$$
$$= \int \sec^{m-1} x (\sec^2 x - 1)^k \sec x \tan x \, dx$$

(c) If there are no scat factors and the power of the tangent is even and positive, convert a tangent-squared factor to a secant-squared factor, then expand and repeat if necessary.

$$\int \tan^n x \, \mathrm{d}x = \int \tan^{n-2} x \overbrace{(\tan^2 x)}^{\text{Convert to secants}} \mathrm{d}x$$
$$= \int \tan^{n-2} x(\sec^2 x - 1) \, \mathrm{d}x$$

(d) If the integral is of the form $\int \sec^m x \, \mathrm{d}x$, where m is odd and positive, use integration by parts, as illustrated in Example 5 in the preceding section.

(e) If none of the first four guidelines applies, try converting to sines and cosines.

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11. **Product-to-Sum Identities**

$$\sin mx \sin nx = \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$$
$$\sin mx \cos nx = \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$$
$$\cos mx \cos nx = \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$$

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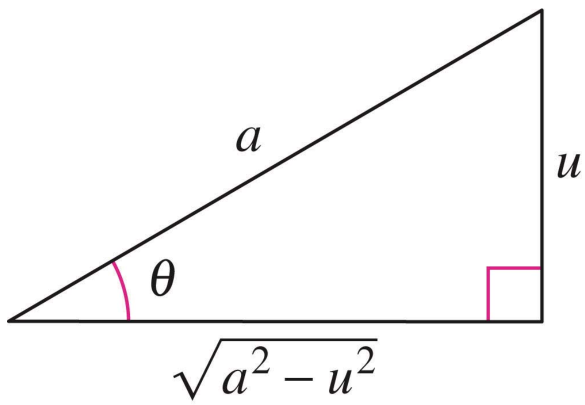
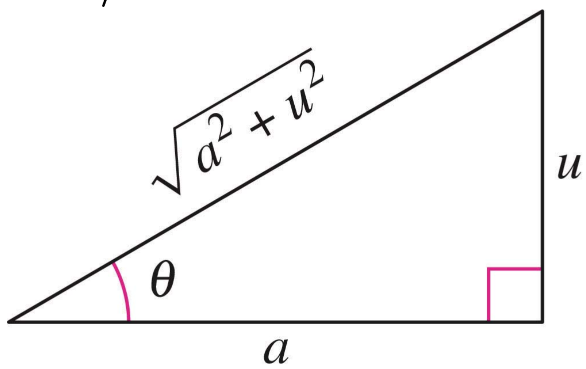
Section 8.4 Trigonometric substitution 53

12. **Trigonometric Substitution** (三角代換) $(a > 0)$

(a) For integrals involving $\sqrt{a^2 - u^2}$, let $u = a \sin\theta$. Then $\sqrt{a^2 - u^2} = a\cos\theta$, where $-\pi/2 \le \theta \le \pi/2$.

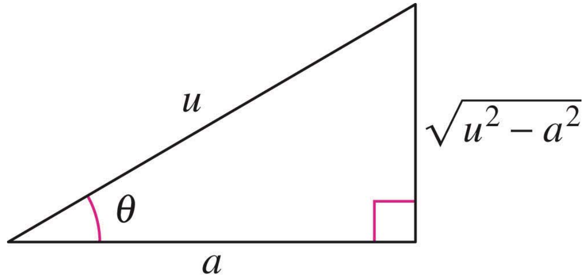


(b) For integrals involving $\sqrt{a^2 + u^2}$, let $u = a \tan \theta$. Then $\sqrt{a^2 + u^2} = a \sec \theta$, where $-\pi/2 \leq \theta \leq \pi/2$.



(c) For integrals involving $\sqrt{u^2 - a^2}$, let $u = a \sec \theta$.

Then $\sqrt{u^2 - a^2} = \begin{cases} a\tan\theta & \text{if } u > a, \text{ where } 0 \le \theta < \pi/2 \\ -a\tan\theta, & \text{if } u < -a, \text{ where } \pi/2 < \theta \le \pi. \end{cases}$



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13. **Special integration formulas** (特殊積分公式) $(a > 0)$

(a) $\int \sqrt{a^2 - u^2}\,\mathrm{d}u = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2 - u^2}\right) + C$

(b) $\int \sqrt{u^2 - a^2}\,\mathrm{d}u = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln\left|u + \sqrt{u^2 - a^2}\right|\right) + C, \quad u > a$

(c) $\int \sqrt{u^2 + a^2}\,\mathrm{d}u = \frac{1}{2}\left(u\sqrt{u^2 + a^2} + a^2 \ln\left|u + \sqrt{u^2 + a^2}\right|\right) + C$

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Section 8.5 Partial fractions 71

14. **Decomposition of $N(x)/D(x)$ into partial fractions** (**把 $N(x)/D(x)$ 分**

(a) **Divide if improper**: If $N(x)/D(x)$ is an improper fraction (that is, if the degree of the numerator is greater than or equal to the degree of the denominator), divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)} = \text{(a polynomial)} + \frac{N_1(x)}{D(x)}$$

where the degree of $N_1(x)$ is less than the degree of $D(x)$. Then apply Steps 2, 3, and 4 to the proper rational expression $N_1(x)/D(x)$.

(b) **Factor denominator**: Completely factor the denominator into factors of the form

$$(px+q)^m \quad \text{and} \quad (ax^2+bx+c)^n$$

where ax^2+bx+c is irreducible.

(c) **Linear factors**: For each factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

(d) **Quadratic factors**: For each factor of the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of n fractions.

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

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15. **Guidelines for solving the basic equation**

Linear Factors

(a) Substitute the roots of the distinct linear factors in the basic equation.

(b) For repeated linear factors, use the coefficients determined in guideline 1 to rewrite the basic equation. Then substitute other convenient values of x and solve for the remaining coefficients.

Quadratic Factors

(a) Expand the basic equation.

(b) Collect terms according to powers of x.

(c) Equate the coefficients of like powers to obtain a system of linear equations involving A, B, C, and so on.

(d) Solve the system of linear equations.

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Section 8.6 Numerical integration 92

16. **The Trapezoidal Rule** Let f be continuous on $[a, b]$. The **Trapezoidal Rule** (梯形法則) for approximating $\int_a^b f(x)\,\mathrm{d}x$ is given by

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right].$$

Moreover, as $n \to \infty$, the right hand side approaches $\int_a^b f(x)\,\mathrm{d}x$. . . . 98

17. **Midpoint Rule** (**中點法則**)

$$\int_a^b f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{2}\right)\Delta x$$

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18. **Integral of $p(x) = Ax^2 + Bx + C$** If $p(x) = Ax^2 + Bx + C$, then

$$\int_a^b p(x)\,\mathrm{d}x = \left(\frac{b-a}{6}\right)\left[p(a) + 4p\left(\frac{a+b}{2}\right) + p(b)\right].$$

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19. **Simpson's Rule** Let f be continuous on $[a, b]$ and let n be an even integer. The **Simpson's Rule** (**辛普森法則**) for approximating

$\int_a^b f(x)\,\mathrm{d}x$ is

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{b-a}{3n}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+4f(x_{n-1})+f(x_n)\right].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,\mathrm{d}x$. . . 107

20. **Errors in the Trapezoidal Rule and Simpson's Rule** If f has a continuous second derivative on $[a, b]$, then the error E in approximating $\int_a^b f(x)\,\mathrm{d}x$ by the Trapezoidal Rule is

$$|E| \leq \frac{(b-a)^3}{12n^2}\left[\max|f''(x)|\right], \quad a \leq x \leq b.$$

error in Trapezoidal Rule (梯形法則的誤差)

Moreover, if f has a continuous fourth derivative on $[a, b]$, then the error E is approximating $\int_a^b f(x)\,\mathrm{d}x$ by Simpson's Rule is

$$|E| \leq \frac{(b-a)^5}{180n^4}\left[\max|f^{(4)}(x)|\right], \quad a \leq x \leq b.$$

error in Simpson's Rule (辛普森法則的誤差)

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Section 8.7 Integration by tables and other integration techniques 113

21. **Substitution for rational functions of sine and cosine** (正弦和餘弦的

For integrals involving rational functions of sine and cosine, the substitution

$$u = \frac{\sin x}{1 + \cos x} = \tan \frac{x}{2}$$

yields

$$\cos x = \frac{1 - u^2}{1 + u^2}, \quad \sin x = \frac{2u}{1 + u^2}, \quad \text{and} \quad \mathrm{d}x = \frac{2\,\mathrm{d}u}{1 + u^2}.$$

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Section 8.8 Improper integrals 128

22. **Improper integrals with infinite integration limits**

(a) If f is continuous on the interval $[a, \infty]$, then

$$\int_a^\infty f(x)\,\mathrm{d}x = \lim_{b \to \infty} \int_a^b f(x)\,\mathrm{d}x.$$

(b) If f is continuous on the interval $[-\infty, b]$, then

$$\int_{-\infty}^{b} f(x)\,\mathrm{d}x = \lim_{a\to-\infty} \int_{a}^{b} f(x)\,\mathrm{d}x.$$

(c) If f is continuous on the interval $[-\infty, \infty]$, then

$$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = \int_{-\infty}^{c} f(x)\,\mathrm{d}x + \int_{c}^{\infty} f(x)\,\mathrm{d}x$$

where c is any real number.

In the first two cases, the **improper integral** (**瑕積分**) **converges** if the limit exists-otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges if either of the improper integral on the right diverges. 130

23. **Improper integrals with infinite discontinuities**

(a) If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{c \to b^-} \int_a^c f(x)\,\mathrm{d}x.$$

(b) If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{c \to a^+} \int_c^b f(x)\,\mathrm{d}x.$$

(c) If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x + \int_c^b f(x)\,\mathrm{d}x.$$

In the first two cases, the improper integral **converges** if the limit exists-otherwise, the improper integral **diverges**. In the third case, the im-

proper integral on the left diverges if either of the improper integral on the right diverges. 142

24. **A special type of improper integral (特殊型態瑕積分)**

$$\int_1^\infty \frac{\mathrm{d}x}{x^p} = \begin{cases} \frac{1}{p-1}, & \text{if } p > 1 \\ \text{diverges}, & \text{if } p \le 1 \end{cases}$$

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