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Chapter **5**

LOGARITHMIC, EXPONENTIAL, AND OTHER TRANSCENDENTAL FUNCTIONS

5.1 Summary

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1. The natural logarithmic function

The natural logarithmic function (自然對數函數) is defined by

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0.$$

The domain of the natural logarithmic function is the set of all positive real numbers..... 6

2. **Properties of the natural logarithmic function** The natural logarithmic function has the following properties.

(a) The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

(b) The function is continuous, increasing, and one-to-one.

(c) The graph is concave downward.

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3. **Logarithmic properties** (對數性質) If a and b are positive numbers and n is rational, then the following properties are true.

(a) $\ln(1) = 0$

(b) $\ln(ab) = \ln a + \ln b$

$$(c) \ln(a^n) = n \ln a$$

$$(d) \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

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4. **e** (**自然對數的底數**) The letter e denotes the positive real number such that

$$\ln e = \int_1^e \frac{1}{t} dt = 1.$$

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5. **Euler's Formula** (**尤拉公式**)

$$e^{ix} = \cos x + i \sin x$$

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6. **Euler's Identity** (**尤拉恆等式**): The most beautiful theorem in math-

ematics.

$$e^{i\pi} + 1 = 0$$

..... 23

7. **Derivative of the natural logarithmic function** Let u be a differentiable function of x .

1. $\frac{d}{dx} [\ln x] = \frac{1}{x}, x > 0$ 2. $\frac{d}{dx} [\ln u] = \frac{1}{u} \frac{du}{dx} = \frac{u'}{u}, u > 0$ 24

8. **Derivative involving absolute value** If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx} \ln |u| = \frac{u'}{u}.$$

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Section 5.2 The natural logarithmic function: integration .. 30

9. **Log Rule for integration** (積分的對數法則) Let u be a differentiable function of x .

1. $\int \frac{1}{x} dx = \ln |x| + C$ 2. $\int \frac{1}{u} du = \ln |u| + C$ 31

10. **Guidelines for integration**

- (a) Learn a basic list of integration formulas. (Including those given in this section, you now have 12 formulas: the Power Rule, the Log Rule, and ten trigonometric rules. By the end of Section ??, this list will have expanded to 20 basic rules.)
- (b) Find an integration formula that resembles all or part of the integrand, and, by trial and error, find a choice of u that will make the integrand conform to the formula.
- (c) If you cannot find a u -substitution that works, try altering the in-

tegrand. You might try a trigonometric identity, multiplication and division by the same quantity, addition and subtraction of the same quantity, or long division. Be creative.

- (d) If you have access to computer software that will find antiderivatives symbolically, use it.

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Table 5.1: Integrals of the six basic trigonometric functions

$\int \sin u \, du = -\cos u + C$	$\int \cos u \, du = \sin u + C$
$\int \tan u \, du = -\ln \cos u + C$	$\int \cot u \, du = \ln \sin u + C$
$\int \sec u \, du = \ln \sec u + \tan u + C$	$\int \csc u \, du = -\ln \csc u + \cot u + C$

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Section 5.3 Inverse functions 43

12. **Inverse function** A function g is the **inverse function** (反函數) of the function f if $f(g(x)) = x$ for each x in the domain of g and $g(f(x)) = x$ for each x in the domain of f . The function g is denoted by f^{-1} (read " f inverse")..... 46
13. **Some important observations about inverse functions**
- (a) If g is the inverse function of f , then f is the inverse function of g .
 - (b) The domain of f^{-1} is equal to the range of f , and the range of f^{-1} is equal to the domain of f .
 - (c) A function need not have an inverse function, but if it does, the inverse function is unique.
- 46

14. Reflective property of inverse functions The graph of f contains the point (a, b) if and only if the graph of f^{-1} contains the point (b, a) 50
15. The existence of an inverse function
- (a) A function has an inverse function if and only if it is one-to-one (一對一).
- (b) If f is strictly monotonic (嚴格單調) on its entire domain, then it is one-to-one and therefore has an inverse function. 54
16. Guidelines for finding an inverse function
- (a) Use Theorem 5.7 to determine whether the function given by $y = f(x)$ has an inverse function.

- (b) Solve for x as a function of y : $x = g(y) = f^{-1}(y)$.
- (c) Interchange x and y . The resulting equation is $y = f^{-1}(x)$.
- (d) Define the domain of f^{-1} as the range of f .
- (e) Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

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17. **Continuity and differentiability of inverse functions** Let f

be a function whose domain is an interval I . If f has an inverse function, then the following statements are true.

- (a) If f is continuous on its domain, then f^{-1} is continuous on its domain.
- (b) If f is increasing on its domain, then f^{-1} is increasing on its domain.
- (c) If f is decreasing on its domain, then f^{-1} is decreasing on its domain.

(d) If f is differentiable on an interval containing c and $f'(c) \neq 0$, then f^{-1} is differentiable at $f(c)$.

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18. **The derivative of an inverse function** Let f be a function that is differentiable on an interval I . If f has an inverse function g , then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0.$$

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Section 5.4 Exponential functions: differentiation and integration

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19. **The natural exponential function** The inverse function of the

natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** (**自然指數函數**) and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

$$y = e^x \quad \text{if and only if} \quad x = \ln y.$$

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20. Operations with exponential functions

Let a and b be any

real numbers.

$$(a) \quad e^a e^b = e^{a+b}$$

$$(b) \quad \frac{e^a}{e^b} = e^{a-b}$$

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21. Properties of the natural exponential function

- (a) The domain of $f(x) = e^x$ is $(-\infty, \infty)$, and the range is $(0, \infty)$.
- (b) The function $f(x) = e^x$ is continuous, increasing, and one-to-one on its entire domain.
- (c) The graph of $f(x) = e^x$ is concave upward on its entire domain.
- (d) $\lim_{x \rightarrow -\infty} e^x = 0$ and $\lim_{x \rightarrow \infty} e^x = \infty$.

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22. Derivatives of the natural exponential function

Let u be a

differentiable function of x .

- (a) $\frac{d}{dx} [e^x] = e^x$
- (b) $\frac{d}{dx} [e^u] = e^u \frac{du}{dx}$

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23. **Integration rules for exponential functions** Let u be a differentiable function of x .

$$1. \int e^x dx = e^x + C \quad 2. \int e^u du = e^u + C \dots\dots\dots 94$$

Section 5.5 Bases other than e and applications..... 96

24. **Exponential function to base a** If a is a positive real number ($a \neq 1$) and x is any real number, then the exponential function to the base a is denoted by a^x and is defined by

$$a^x = e^{(\ln a)x}.$$

If $a = 1$, then $y = 1^x = 1$ is a constant function..... 98

25. **Logarithmic function to base a** If a is a positive real number ($a \neq 1$) and x is any positive real number, then the logarithmic function

to the base a is denoted by $\log_a x$ and is defined as

$$\log_a x = \frac{1}{\ln a} \ln x.$$

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26. Properties of inverse functions

(a) $y = a^x$ if and only if $x = \log_a y$.

(b) $a^{\log_a x} = x$, for $x > 0$.

(c) $\log_a a^x = x$, for all x .

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27. Derivatives for bases other than e

Let a be a positive real

number ($a \neq 1$) and let u be a differentiable function of x .

$$1. \frac{d}{dx} [a^x] = (\ln a)a^x$$

$$2. \frac{d}{dx} [a^u] = (\ln a)a^u \frac{du}{dx}$$

$$3. \frac{d}{dx} [\log_a x] = \frac{1}{(\ln a)x}$$

$$4. \frac{d}{dx} [\log_a u] = \frac{1}{(\ln a)u} \frac{du}{dx}$$

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28. The Power Rule for real exponents

Let n be any real number

and let u be a differentiable function of x .

$$(a) \frac{d}{dx} [x^n] = nx^{n-1}$$

$$(b) \frac{d}{dx} [u^n] = nu^{n-1} \frac{du}{dx}$$

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29. A limit involving e

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x}\right)^x = e$$

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30. Summary of compound interest formulas (複利公式)

Let P = amount of deposit, t = number of years, A = balance after t years, r = annual interest rate (decimal form), and n = number of compoundings per year.

(a) Compounded n times per year: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

(b) Compounded continuously (連續複利): $A = Pe^{rt}$

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Section 5.6 Indeterminate forms and L'Hôpital's Rule 120

31. The Extended Mean Value Theorem (廣義均值定理) If f and

g are differentiable on an open interval (a, b) and continuous on $[a, b]$ such that $g'(x) \neq 0$ for any x in (a, b) , then there exists a point c in (a, b)

such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}.$$

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32. **L'Hôpital's Rule** (羅比達法則) Let f and g be functions that are differentiable on an open interval (a, b) containing c , except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b) , except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form $0/0$, then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies if the limit of $f(x)/g(x)$ as x approaches c produces anyone of

the indeterminate forms $\infty/\infty, (-\infty)/\infty, \infty/(-\infty)$ or $(-\infty)/(-\infty)$.
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Section 5.7 Inverse trigonometric functions: differentiation 140

33. Inverse trigonometric functions (反三角函數)

Function	Domain	Range	
$y = \arcsin x$ iff $\sin y = x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$	
$y = \arccos x$ iff $\cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$	
$y = \arctan x$ iff $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$. 142
$y = \operatorname{arccot} x$ iff $\cot y = x$	$-\infty < x < \infty$	$0 < y < \pi$	
$y = \operatorname{arcsec} x$ iff $\sec y = x$	$ x \geq 1$	$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$	
$y = \operatorname{arccsc} x$ iff $\csc y = x$	$ x \geq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$	

34. Properties of inverse trigonometric functions If $-1 \leq x \leq$

1 and $-\pi/2 \leq y \leq \pi/2$, then

$$\sin(\arcsin x) = x \quad \text{and} \quad \arcsin(\sin y) = y.$$

If $-\infty < x < \infty$ and $-\pi/2 < y < \pi/2$, then

$$\tan(\arctan x) = x \quad \text{and} \quad \arctan(\tan y) = y.$$

If $|x| \geq 1$ and $0 \leq y < \pi/2$ or $\pi/2 < y \leq \pi$, then

$$\sec(\operatorname{arcsec} x) = x \quad \text{and} \quad \operatorname{arcsec}(\sec y) = y.$$

Similar properties hold for the other inverse trigonometric functions. 146

35. Derivatives of inverse trigonometric functions

Let u

be a differentiable function of x .

$$\frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

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36. Elementary function

An elementary function (基本函数)

is a function from the following list or one that can be formed as the sum, product, quotient, or composition of functions in the list.

Algebraic Functions	Transcendental Functions	
Polynomial functions	Logarithmic functions	
Rational functions	Exponential functions160
Functions involving radicals	Trigonometric functions	
	Inverse trigonometric functions	

37. Basic differentiation rules for elementary functions

$$\begin{array}{lll}
1. \frac{d}{dx} [cu] = cu' & 2. \frac{d}{dx} [u \pm v] = u' \pm v' & 3. \frac{d}{dx} [uv] = uv' + vu' \\
4. \frac{d}{dx} \left[\frac{u}{v} \right] = \frac{vu' - uv'}{v^2} & 5. \frac{d}{dx} [c] = 0 & 6. \frac{d}{dx} [u^n] = nu^{n-1}u' \\
7. \frac{d}{dx} [x] = 1 & 8. \frac{d}{dx} [|u|] = \frac{u}{|u|}(u'), \quad u \neq 0 & 9. \frac{d}{dx} [\ln u] = \frac{u'}{u} \\
10. \frac{d}{dx} [e^u] = e^u u' & 11. \frac{d}{dx} [\log_a u] = \frac{u'}{(\ln a)u} & 12. \frac{d}{dx} [a^u] = (\ln a)a^u u' \\
13. \frac{d}{dx} [\sin u] = (\cos u)u' & 14. \frac{d}{dx} [\cos u] = -(\sin u)u' & 15. \frac{d}{dx} [\tan u] = (\sec^2 u)u' \\
16. \frac{d}{dx} [\cot u] = -(\csc^2 u)u' & 17. \frac{d}{dx} [\sec u] = (\sec u \tan u)u' & 18. \frac{d}{dx} [\csc u] = -(\csc u \cot u)u' \\
19. \frac{d}{dx} [\arcsin u] = \frac{u'}{\sqrt{1-u^2}} & 20. \frac{d}{dx} [\arccos u] = \frac{-u'}{\sqrt{1-u^2}} & 21. \frac{d}{dx} [\arctan u] = \frac{u'}{1+u^2} \\
22. \frac{d}{dx} [\operatorname{arccot} u] = \frac{-u'}{1+u^2} & 23. \frac{d}{dx} [\operatorname{arcsec} u] = \frac{u'}{u\sqrt{u^2-1}} & 24. \frac{d}{dx} [\operatorname{arccsc} u] = \frac{-u'}{u\sqrt{1-u^2}}
\end{array}$$

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Section 5.8 Inverse trigonometric functions: integration ... 160**38. Identities involving inverse trigonometric functions**

$$\arcsin x + \arccos x = \frac{1}{2}\pi, \quad |x| \leq 1$$

$$\arctan x + \operatorname{arccot} x = \frac{1}{2}\pi, \quad |x| \in \mathbb{R}$$

$$\operatorname{arcsec} x + \operatorname{arccsc} x = \frac{1}{2}\pi, \quad |x| \geq 1$$

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39. Integrals involving inverse trigonometric functionsLet u

be a differentiable function of x , and let $a > 0$.

$$\mathbf{1.} \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C \quad \mathbf{2.} \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C \quad \mathbf{3.}$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C \dots\dots\dots 163$$

40. Basic integration rules ($a > 0$)

-
1. $\int k f(u) \, du = k \int f(u) \, du$ 2. $\int [f(u) \pm g(u)] \, du = \int f(u) \, du \pm \int g(u) \, du$
3. $\int du = u + C$ 4. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln |u| + C$ 6. $\int e^u \, du = e^u + C$
7. $\int a^u \, du = \left(\frac{1}{\ln a}\right) a^u + C$ 8. $\int \sin u \, du = -\cos u + C$
9. $\int \cos u \, du = \sin u + C$ 10. $\int \tan u \, du = -\ln |\cos u| + C$
11. $\int \cot u \, du = \ln |\sin u| + C$ 12. $\int \sec u \, du = \ln |\sec u + \tan u| + C$.169
13. $\int \csc u \, du = -\ln |\csc u + \cot u| + C$ 14. $\int \sec^2 u \, du = \tan u + C$
15. $\int \csc^2 u \, du = -\cot u + C$ 16. $\int \sec u \tan u \, du = \sec u + C$
17. $\int \csc u \cot u \, du = -\csc u + C$ 18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

Section 5.9 Hyperbolic functions 172

41. The hyperbolic functions (雙曲函數)

$$\begin{aligned} \sinh x &= \frac{e^x - e^{-x}}{2} & \operatorname{csch} x &= \frac{1}{\sinh x}, \quad x \neq 0 \\ \cosh x &= \frac{e^x + e^{-x}}{2} & \operatorname{sech} x &= \frac{1}{\cosh x} \\ \tanh x &= \frac{\sinh x}{\cosh x} & \operatorname{coth} x &= \frac{1}{\tanh x}, \quad x \neq 0 \end{aligned}$$

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42. Hyperbolic identities (雙曲函數恆等式)

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= 1 & \sinh(x + y) &= \sinh x \cosh y + \cosh x \sinh y \\ \tanh^2 x + \operatorname{sech}^2 x &= 1 & \sinh(x - y) &= \sinh x \cosh y - \cosh x \sinh y \\ \operatorname{coth}^2 x - \operatorname{csch}^2 x &= 1 & \cosh(x + y) &= \cosh x \cosh y + \sinh x \sinh y \end{aligned}$$

$$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\sinh^2 x = \frac{-1 + \cosh 2x}{2} \quad \cosh^2 x = \frac{1 + \cosh 2x}{2}$$

$$\sinh 2x = 2 \sinh x \cosh x \quad \cosh 2x = \cosh^2 x + \sinh^2 x$$

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43. Derivatives and integrals of hyperbolic functions

Let u be

a differentiable function of x .

$$\begin{array}{ll} \frac{d}{dx} [\sinh u] = (\cosh u)u' & \int \cosh u \, du = \sinh u + C \\ \frac{d}{dx} [\cosh u] = (\sinh u)u' & \int \sinh u \, du = \cosh u + C \\ \frac{d}{dx} [\tanh u] = (\operatorname{sech}^2 u)u' & \int \operatorname{sech}^2 u \, du = \tanh u + C \end{array}$$

$$\frac{d}{dx} [\coth u] = -(\operatorname{csch}^2 u)u' \quad \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$\frac{d}{dx} [\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u' \quad \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$\frac{d}{dx} [\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u' \quad \int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

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44. Inverse hyperbolic functions (反雙曲函數)

Function

Domain

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$(-\infty, \infty)$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$$

$$[1, \infty)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$$

$$(-1, 1)$$

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$$\coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1}$$

$$(-\infty, -1) \cup (1, \infty)$$

$$\operatorname{sech}^{-1} x = \ln \frac{1 + \sqrt{1 - x^2}}{x}$$

$$(0, 1]$$

$$\operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|} \right)$$

$$(-\infty, 0) \cup (0, \infty)$$

45. Differentiation and integration involving inverse hyperbolic functions

Let u be a differentiable function of x .

$$\begin{aligned} \frac{d}{dx} [\sinh^{-1} u] &= \frac{u'}{\sqrt{u^2 + 1}} & \frac{d}{dx} [\cosh^{-1} u] &= \frac{u'}{\sqrt{u^2 - 1}} \\ \frac{d}{dx} [\tanh^{-1} u] &= \frac{u'}{1 - u^2} & \frac{d}{dx} [\coth^{-1} u] &= \frac{u'}{1 - u^2} \\ \frac{d}{dx} [\operatorname{sech}^{-1} u] &= \frac{-u'}{u\sqrt{1 - u^2}} & \frac{d}{dx} [\operatorname{csch}^{-1} u] &= \frac{-u'}{|u|\sqrt{1 + u^2}} \end{aligned}$$

$$\begin{aligned} \int \frac{du}{\sqrt{u^2 \pm a^2}} &= \ln(u + \sqrt{u^2 \pm a^2}) + C \\ \int \frac{du}{a^2 - u^2} &= \frac{1}{2a} \ln \left| \frac{a + u}{a - u} \right| + C \\ \int \frac{du}{u\sqrt{a^2 \pm u^2}} &= -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C \end{aligned}$$

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