CALCULUS (I)

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Lectures

- Lecture: Szu-Chi Chung (鍾思齊)
  - Office: 理 SC 2002-4
  - Office hour: Mon. 16:00~18:00 and Wed. 16:00~18:00
- T.A.: 林暐庭、黃聖勛
  - Office: 理 SC2003-3
  - TA hour: Tue. 14:00~16:00
- Class hours: Wed. (09:10-12:00)
  - Classroom: 工EC 1001
- Facebook: https://www.facebook.com/groups/381395079826853

Textbook and requirement

▸ Textbook *: Calculus, 12th Ed (Metric Version)*

   ▸ Authors: Larson, Ron and Edwards, Bruce H

   ▸ https://www.tunghua.com.tw/portal_b10_cnt_page.php?button_num=b10&folder_id=666&cnt_id=10267

▸ Graphical tools

   ▸ https://www.wolframalpha.com/

   ▸ https://wolfreealpha.gitlab.io/

   ▸ Mobile app

Textbook and requirement

▸ Resources provided by authors

   ▸ https://www.larsoncalculus.com/calc11/

▸ For the odd number of exercises of each chapter, the solution is at the end of the book

▸ Prerequisite

   ▸ If you are unfamiliar with precalculus, read Chapter P first

      ▸ Graphs and models

      ▸ Linear models and rates of change

      ▸ Functions and their graphs

      ▸ Review of trigonometric functions

▸ The assignment and related material will be available on the course webpage

   ▸ https://phonchi.github.io/nsysu-calculus1/

Resources

- Visual introduction to the core ideas in calculus
  - https://www.3blue1brown.com/topics/calculus
- Precalculus
  - 寫給高中生的微積分簡介-第五版
- Calculus
  - 微積分(黃文璋老師)
  - 微積分總棟員
  - 微積分(嚴國勇老師)
  - 微積分(朱樺老師)
  - 微積分(李國偉老師)

Grading policy

▶ Grading

  ▶ Weekly Homework 20%

  ▶ Participants: 5% (participates at least 10 times can get the full score)

  ▶ Take home Quiz: 5% (2 times)

  ▶ Midterm exam: 30%

  ▶ Final exam: 40%

▶ Midterm

  ▶ Will be held on 2023/10/25 atエEC 1001

▶ Final:

  ▶ Will be held on 2023/12/20 at エEC 1001

Relate to other courses

- Related courses
  - Differential equations
  - General physics (I)(II)
  - Circuit theory (I)(II)
  - Electromagnetic theory (I)(II) – Vector Calculus
  - Signals and systems
  - Probability, Complex variable….
- Related fields
  - Machine learning, statistics
  - Applied mathematics
  - …

What is the difference between precalculus?

▶ Current semester

  ▶ Formal discussion of limit

  ▶ Implicit differentiation

  ▶ Involve logarithm, exponential and other transcendental function

  ▶ Integration techniques and improper integral

▶ Next semester

  ▶ Functions of Several Variables

  ▶ Multiple integration

  ▶ Vector calculus…

WolframAlpha

▸ WolframAlpha is a computational knowledge engine or answer engine developed by Wolfram

  ▸ It is an online service that answers queries directly by computing the answer from "curated data", rather than providing a list of documents or web pages that might contain the answer, as a search engine might

  ▸ WolframAlpha has been used to power some searches in the Microsoft Bing and DuckDuckGo. For factual question answering, it is sometimes queried by Apple's Siri, Amazon Alexa and ChatGPT for math and science queries

  ▸ Can be used for math, physics, chemistry, earth science, engineering, weather, ….

Some examples you may encounter this semester

- Evaluate
  - x^2 - 2 x at x = 10
  - x^2 - 2 x at x=3.9, 3.99,3.999,4.001,4.01,4.1 (Generate a table)

- Plotting
  - plot x^2 from -10 to 10
  - parametric plot {sin(t), cos(t)} t from 0 to 2 pi
  - polar plot sin(2t) t from 0 to 2 pi

- Domain and range
  - domain of f(x) = x/(x^2-1)
  - range of x^2 - x - 1

- ▸ Limits
  - ▸ limit of (sin x)/x as x->0
  - ▸ limit of (1+1/n)^n as n->infinity
  - ▸ limit of x/|x| as x->0+
- ▸ Continuous
  - ▸ Is f(x)=x sin(x^2) continuous over the reals?
  - ▸ is tan(x) continuous at pi?
  - ▸ discontinuities (x^3+8)/(x^3+3x^2-4x-12)
- ▸ Derivative
  - ▸ derivative of x^4 sin x
  - ▸ derivative of x^4 sin x at x=2
  - ▸ second derivative of sin(2x) (High order derivative)
  - ▸ differentiate x^2 - 4y^2 = 1 with respect to x (Implicit derivative)
- ▸ Differentiability
  - ▸ is f(x) = sin^2(x) differentiable?

▸ is y = abs(x) differentiable

- Curve
  - tangent line of y = x sin^2(x) at x = 2
  - critical points of x^4 - 6 x^3 + x + 10
  - inflection points of x^4 - 6 x^3 + x + 10
  - local minima of 24 x^5 - 75 x^4 - 200 x^3 + 450 x^2 + 1080 x
  - asymptotes of (2x^3 + 4x^2 - 9)/(3 - x^2)
- Newton's method
  - find root of f (x) = 2x^3 + x^2 − x + 1 using Newton's method
- Integrate
  - integrate x^2 sin^3 x dx (indefinite integral)
  - integrate sin x dx from 0 to pi (definite Integrals)
  - integrate sinx/x dx, from 0 to infinity (improper integral)
- Application
  - area between 1 - 2 x + x^2 and 6 x - x^2 from -1 to 1
  - arc length of y=x^2 from x=0 to 1

- Sequence
  - 1, 4, 9, 16, 25, ... (Sequence Recognition)
  - 1+1/2+1/3+1/4+...
  - limit (1+1/n)^n as n->infinity
- Infinite sum
  - sum (3/4)^i, i=0..n
  - sum (3/4)^i, i=0..infinity
  - sum 1/n^2 (infinite sum)
- Sum convergence
  - convergence of n^(-2)
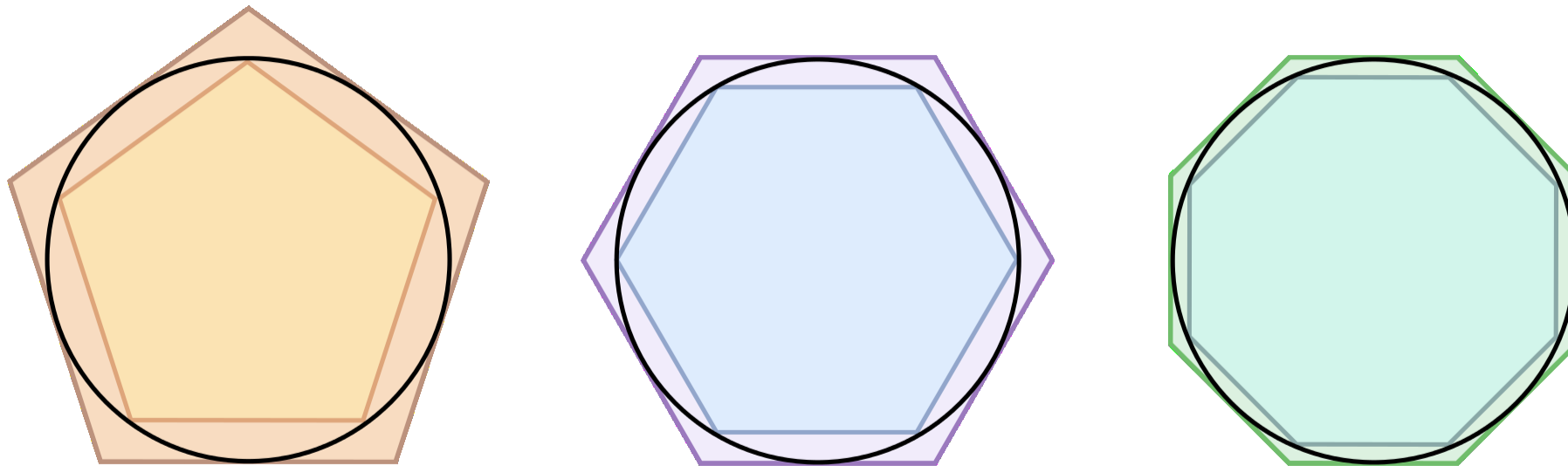 - sum convergence of n^(-2)
  - does the sum of 1/(n!) converge
- Radius of convergence
  - radius of convergence of (3x-2)^n/n

The origin of calculus

▸ The method of exhaustion

▸ Is a method of finding the area of a shape by inscribing inside it a sequence of polygons whose areas converge to the area of the containing shape

▸ From Eudoxus to Archimedes (5th ~3th century BC)

The origin of calculus

- At 16th-century, the following problems become more and more important
  1. Analyze the velocity of an accelerating object
  2. Analyze the slope of a curve (tangent and normal line)
  3. Find the minimum and maximum of a function
  4. Analyze the area under a curve and find the arc length

- Modern calculus was developed in 17th-century Europe by <u>Isaac Newton</u> and <u>Gottfried Wilhelm Leibniz</u> (independently of each other, first publishing around the same time)