If the limit does not exist or has an infinite limit, you should point it out. In addition, do not use the L'Hôpital's rule to solve the limit problem.

1. (16%) Find the following limit

   (a) $\lim_{x \to 1} \frac{x^3-3x+2}{x^3-x^2-x+1}$

   (b) $\lim_{x \to \infty} x(\sqrt{x^2+1}-x)$

   (c) $\lim_{\theta \to 0} \frac{1-\cos(\theta)}{\theta^2}$

   (d) $\lim_{x \to 3} \frac{\sqrt{3x+1}}{x-3}$

2. (8%) Considering the following function.

$$f(x) = \begin{cases} |x| \sin(\frac{1}{x}), & \mathrm{x} \neq 0 \\ 0, & \mathrm{x} = 0 \end{cases}$$

(a) Is $f(x)$ continuous at $x = 0$? Explain your answer.
(b) Is $f(x)$ differentiable at $x = 0$? Explain your answer.

3. (8%) Proof that there is only one intersect point between $f(x) = 2x - 2$ and $g(x) = \cos x$. (Hint: use the intermediate value theorem and mean value theorem/Rolle's theorem)

4. (15%) Remember that you can solve the derivative using the definition or the differentiation rule for the following question.

(a) Find the following limit. $\lim_{x \to 0} \frac{cos(\pi+x)+1}{x}$

(b) Find the derivative of $f(x) = \frac{x^3+3x-1}{x+1}$

(c) Let $f(x) = xcos(x) - tan(x) + 2\pi$, find $f''(x)$

5. (8%) Given $x^2 + \frac{y^2}{4} = 1$, find all the tangent lines of the graph that pass the point (3,0) (Note (3,0) is not on the graph).

6. (15%) Let $f(x) = \frac{x^3}{(x+2)^2}$

(a) Find the critical numbers and the possible points of inflection of $f(x)$

(b) Find the open intervals on which f is increasing or decreasing

(c) Find the open intervals of concavity

(d) Find all the asymptotes (Vertical/horizontal/Slant)

(e) Sketch the graph of $f(x)$ (Label any intercepts, relative extrema, points of inflection,and asymptotes)

7. (15%) Remember the meaning and the definition of definite integral when solving the following question

(a) $\int \frac{2+t+t^3}{\sqrt{t}} dt$

(b) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (t^3 + t^6 \tan(t))\, dt$

(c) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} (\frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \ldots + \frac{1}{\sqrt{n+n}})$

8. (9%) Considering the function $f(x) = cos(x) + 2cos(2x) + \cdots + ncos(nx)$. Proof that there exists at least one root between $(0, \pi)$ (Hint: Let $F(x) = \int_0^x f(t)dt$ and use the fundamental theorem of calculus as well as Rolle's theorem.)

9. (6%) Evaluate $\int_{\frac{1}{4}}^{1} \frac{\sqrt{1-\sqrt{x}}}{\sqrt{x}} dx$