1. (20%) Determine whether the series converges absolutely or conditionally, or diverges. In addition, please indicate the test you use.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n(2n-1)}{3n+4}$

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+1}+\sqrt{n^3}}$

   (c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+1}$

   (d) $\sum_{n=3}^{\infty} \frac{(-1)^n}{n(\ln n)[\ln(\ln n)]^2}$

   (e) $\sum_{n=1}^{\infty} (\sqrt[n]{n} - 1)$

2. (8%) Consider the function.

$$f(\mathrm{x}) = \frac{1}{2x-1}, \mathrm{x} \neq \frac{1}{2}$$

(a) Find the power series expansion of $p(\mathrm{x})$ of f expand at the point $\frac{1}{3}$ and determine its interval of convergence.

(b) Write $p(\mathrm{x}) = \sum_{n=0}^{\infty} a_n (x - \frac{1}{3})^n$. Is $\sum_{n=0}^{\infty} a_n (\frac{2}{3})^n = f(1) = 1$? and why?

3. (10%)

(a) Find the Maclaurin series for $\arccos(\mathrm{x})$.

(b) Find the radius and interval of convergence of the Maclaurin series for $\arccos(\mathrm{x})$

4. (8%) Use a power series to approximate $\int_0^1 \sin(x^2)dx$ with an error of less than 0.001.

5. (9%) Evaluate the following expression (Try to use the Basic series of Taylor series and notice that the power series is a continuous function).

   (a) $1 - \frac{\pi^2}{4^2 \times 2!} + \frac{\pi^4}{4^4 \times 4!} - \frac{\pi^6}{4^6 \times 6!} + \cdots$

   (b) $\frac{1}{\sqrt{3}} - \frac{1}{3(\sqrt{3})^3} + \frac{1}{5(\sqrt{3})^5} - \frac{1}{7(\sqrt{3})^7} + \cdots$

   (c) $\lim_{x \to 0} \frac{\tan(x) - \sin(x)}{x^2}$

6. (8%) Let $f(\mathrm{x}) = x^6 e^{x^3}$. Try to evaluate the high order derivative $f^{(60)}(0)$.

7. (8%) Find the area of the region which is inside the circle $r = 6\cos(\theta)$ and outside the cardioid $r = 2(1 + \cos(\theta))$. (Both are represented in polar coordinates).

8. (5%) Find the arc length of $r = e^{\theta}$ from $\theta = 0$ to $\theta = 2\pi$.

9. (12%) Classify the following surface. If it is a quadratic surface, you should further classify it into six basic types of quadratic surface.

(a) $\mathrm{z} = x^2 + 3y^2$

(b) $x^2 + y^2 - 2z = 0$

(c) $r^2 = z^2 + 2$ (this representation is in cylindrical coordinates)

(d) $\rho = 4\sec(\Phi)$ (this representation is in spherical coordinates)

10. (12%) Evalauate the following expression.

(a) $\lim_{t\to 1} \sqrt{t}\boldsymbol{i} + \frac{\ln t}{t^2-1}\boldsymbol{j} + \frac{1}{t-1}\boldsymbol{k}$

(b) $\lim_{t\to 0} \frac{sin2t}{t}\boldsymbol{i} + e^{-t}\boldsymbol{j} + 5\boldsymbol{k}$

(c) Let $\mathbf{r}(\mathrm{t}) = 3\mathrm{t}\mathbf{i} + (\mathrm{t}-1)\mathbf{j}$, $\mathbf{u}(\mathrm{t}) = \mathrm{t}\mathbf{i} + t^2\mathbf{j} + \frac{2}{3}t^3\boldsymbol{k}$, find $\frac{d}{dt}[\boldsymbol{r}(t)\cdot\boldsymbol{u}(t)]$

(d) $\int(3\sqrt{t}\boldsymbol{i} + \frac{2}{t}\boldsymbol{j} + 6\boldsymbol{k})dt$