1. (20%) Find the following limit. (If the limit does not exist, you should point it out).

   Hint: Change of variables may be useful here

   (a) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

   (b) $\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$

   (c) $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$

   (d) $\lim_{(x,y)\to(0,0)} \frac{x^3+xy^2}{4x^2y-2y^3}$

   (e) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

2. (16%) Evaluate the following problems

(a) Let $f(x,y) = \int_y^x \sin(t^2)dt,$ find f_x and f_y

(b) Let $f(x,y) = x\sin(\mathrm{y}) + \mathrm{y}e^{xy}$, find all the second partial derivatives of f

(c) Let $z = f(x,y) = x^2 + y^2, x = s + t, y = s - t,$ find $\frac{\partial \mathrm{z}}{\partial s}$ and $\frac{\partial \mathrm{z}}{\partial \mathrm{t}}$

(d) Find an equation of the tangent plance to the surface $9x^2 + y^2 + 4z^2 = 25$ at $(0,-3,2)$

3. (6%) Let $f(\mathrm{x},\mathrm{y}) = 2022 - \frac{x^2}{4} - \frac{y^2}{2}$, express the limit $\lim_{\mathrm{t}\to 0} \frac{f(1+2t,2+t)-f(1,2)}{\mathrm{t}}$ as the directional derivative of f and evaluate the value of the limit.

4. (8%) Find the critical points of $f(\mathrm{x},\mathrm{y}) = x^3 + y^2 - 2xy + 7x - 8y + 2.$ Which of them give rise to maximum values, minimum values and saddle points?

5. (6%) Find the minimum and maximum distance from the curve $x^2 + xy + y^2 = 1$ to the origin point $(0,0)$.

6. (20%) Evaluate the following expression

(a) $\int_0^1 \int_y^1 \frac{\sin(x)}{x} dxdy$

(b) $\int_0^2 \int_0^{\sqrt{4-x^2}} \sin(\sqrt{x^2+y^2})\, dydx$

(c) $\int_0^{\frac{\pi}{2}} \int_1^2 x^2 \sin(y)\, dxdy$

(d) $\int_0^1 \int_0^{1+\sqrt{y}} \int_0^{xy} y\ \ dzdxdy$

(e) $\int_1^2 \int_{2u-2}^{u} e^{(v-u+1)^2}\, dvdu$

7. (6%) Find the area of the surface given by $z = f(\mathrm{x,y}) = 9 - y^2$ that lies above the region *R* where *R* is a triagle with vertices $(-3,3), (0,0), (3,3)$

8. (6%) Find the volume of the solid inside both $x^2 + y^2 + z^2 = 36$ and $(x-3)^2 + y^2 = 9$.

9. (6%) Evaluate $\int\int\int_Q \frac{z}{\sqrt{x^2+y^2+z^2}} dV$ where Q is a solid region inside the sphere $x^2 + y^2 + z^2 = 9$ and above xy-plane.

10. (6%) Use a change of variables to find the volume of the solid region lying below the surface $\mathrm{z} = f(\mathrm{x,y}) = \frac{x}{1+x^2y^2}$ and above the plane region R where R is a region bounded by $xy = 5, xy = 1, x = 1, x = 5$.