Assignment 9

1. Find the f_x, f_y, f_z.

   (a) $f(x,y) = 7ye^{y/x}$

   (b) $f(x,y) = \arccos(xy)$

   (c) $f(x,y) = \ln\dfrac{x+y}{x-y}$

   (d) $f(x,y,z) = \sqrt{3x^2+y^2-2z^2}$

   (e) $f(x,y,z) = \dfrac{xy}{x+y+z}$

2. Find the total differential.

   (a) $f(x,y) = e^{-x}\tan y$

   (b) $f(x,y,z) = \dfrac{x+y}{z-3y}$

sol:

1. (a)

$$\begin{aligned} f(x,y) &= 7ye^{y/x} = 7ye^{yx^{-1}} \\ f_x &= 7ye^{yx^{-1}}(-yx^{-2}) = \frac{-7y^2}{x^2}e^{y/x} \\ f_y &= 7e^{y/x} + 7\left(\frac{1}{x}\right)ye^{y/x} = 7e^{y/x}\left(1+\frac{y}{x}\right) \end{aligned}$$

   (b)

$$\begin{aligned} f(x,y) &= \arccos xy \\ f_x &= \frac{-y}{\sqrt{1-x^2y^2}} \\ f_y &= \frac{-x}{\sqrt{1-x^2y^2}} \end{aligned}$$

   (c)

$$\begin{aligned} f(x,y) &= \ln\frac{x+y}{x-y} = \ln(x+y) - \ln(x-y) \\ f_x &= \frac{1}{x+y} - \frac{1}{x-y} = \frac{-2y}{(x+y)(x-y)} \\ f_y &= \frac{1}{x+y} + \frac{1}{x-y} = \frac{2x}{(x+y)(x-y)} \end{aligned}$$

   (d)

$$\begin{aligned} f(x,y,z) &= \sqrt{3x^2+y^2-2z^2} \\ f_x &= \frac{6x}{2\sqrt{3x^2+y^2-2z^2}} = \frac{3x}{\sqrt{3x^2+y^2-2z^2}} \\ f_y &= \frac{2y}{2\sqrt{3x^2+y^2-2z^2}} = \frac{y}{\sqrt{3x^2+y^2-2z^2}} \\ f_z &= \frac{-4z}{2\sqrt{3x^2+y^2-2z^2}} = \frac{-2z}{\sqrt{3x^2+y^2-2z^2}} \end{aligned}$$

(e)

$$
\begin{aligned}
f(x,y,z) &= \frac{xy}{x+y+z} \\
f_x &= \frac{(x+y+z)y - xy}{(x+y+z)^2} = \frac{y^2+yz}{(x+y+z)^2} \\
f_y &= \frac{(x+y+z)x - xy}{(x+y+z)^2} = \frac{x^2+xz}{(x+y+z)^2} \\
f_z &= \frac{(x+y+z)(0) - xy}{(x+y+z)^2} = \frac{-xy}{(x+y+z)^2}
\end{aligned}
$$

2. (a)

$$
\begin{aligned}
z &= e^{-x}\tan y \\
dz &= -e^{-x}\tan y \; dx + e^{-x}\sec^2 y \; dy
\end{aligned}
$$

(b)

$$
\begin{aligned}
w &= \frac{x+y}{z-3y} \\
dw &= \frac{1}{z-3y}\; dx + \frac{3x+z}{(z-3y)^2}\; dy - \frac{x+y}{(z-3y)^2}\; dz
\end{aligned}
$$