Assignment 8

1. Find the domain and range of the function.

   (a) $f(x,y) = \sqrt{9 - 6x^2 + y^2}$ (b) $f(x,y) = \ln(xy - 6)$

2. Describe and sketch the graph of the level surface $f(x,y,z) = c$ at the given value of c.

$$f(x,y,z) = x^2 + \frac{1}{4}y^2 - z \ , \ c = 1$$

3. Use polar coordinates and L'Hopital's rule to find the limit.

$$\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2)$$

4. Discuss the continuity of the function.

   (a) $f(x,y,z) = \dfrac{\sin z}{e^x + e^y}$ (b) $f(x,y) = \begin{cases} \dfrac{\sin x^2 + y^2}{x^2 - y^2} & , \ x^2 \neq y^2 \\ 1 & , \ x^2 = y^2 \end{cases}$

sol:

1. (a)

$$\begin{aligned} \text{Domain}: \ 9 - 6x^2 + y^2 &\geq 0 \\ 6x^2 - y^2 &\leq 9 \\ \frac{x^2}{3/2} - \frac{y^2}{9} &\leq 1 \\ \text{Range}: \ 0 \leq z &\leq 3 \end{aligned}$$

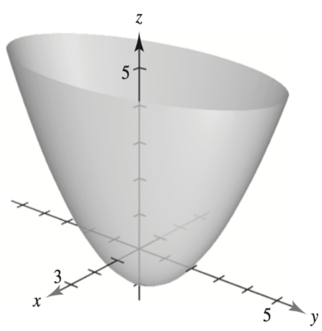
   (b)

$$\begin{aligned} \text{Domain}: \ xy - 6 &> 0 \\ xy &> 6 \end{aligned}$$

$$\text{Range}: \text{ all real numbers}$$

2.

$$\begin{aligned} f(x,y,z) &= x^2 + \frac{1}{4}y^2 - z \\ c &= 1 \\ 1 &= x^2 + \frac{1}{4}y^2 - z \\ &\text{Elliptic paraboloid} \\ &\text{Vertex}: \ (0,0,-1) \end{aligned}$$



3.

$$
\begin{aligned}
x^2+y^2 &= r^2 \\
\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) &= \lim_{r\to 0} r^2 \ln r^2 \\
&= \lim_{r\to 0^+} 2r^2 \ln r \\
\text{By L'Hopital's Rule, } \lim_{r\to 0^+} 2r^2 \ln r &= \lim_{r\to 0^+} \frac{2\ln r}{1/r^2} \\
&= \lim_{r\to 0^+} \frac{2/r}{-2/r^3} \\
&= \lim_{r\to 0^+} (-r^2) \\
&= 0
\end{aligned}
$$

4. (a)

$$f(x,y,z) = \frac{\sin z}{e^x + e^y}$$

Continuous everywhere

(b)

For $x^2 \neq y^2$, the function is clearly continuous.

For $x^2 \neq y^2$, let $z = x^2 - y^2$

Then $\lim_{z\to 0} \frac{\sin z}{z} = 1$

implies that f is continuous for all x, y.