Assignment 7

1. Sketch the plane curve represented by the vector-valued function and give the orientation of the curve.
$$r(t) = \cos\theta\mathbf{i} + 3\sin\theta\mathbf{j}$$

2. Find the limit (if it exists).
$$\lim_{t\to 0}\left(e^t\mathbf{i} + \frac{\sin t}{t}\mathbf{j} + e^{-t}\mathbf{k}\right)$$

3. Find the open interval(s) on which the curve given by the vector- valued function is smooth.
$$r(t) = t\mathbf{i} - 3t\mathbf{j} + \tan t\mathbf{k}$$

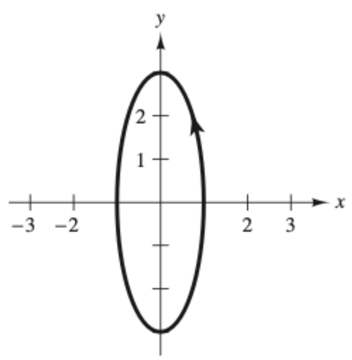
4. Find the indefinite integral.

   (a)
$$\int (e^t\mathbf{i} + \mathbf{j} + t\cos t\mathbf{k})\, dt$$

   (b)
$$\int_0^{\frac{\pi}{4}} [(\sec t\tan t)\mathbf{i} + (\tan t)\mathbf{j} + (2\sin t\cos t)\mathbf{k}]\, dt$$

sol:

1.
$$x = \cos\theta, y = 3\sin\theta$$
$$x^2 + \frac{y^2}{9} = 1, \text{Ellipse}$$



2.
$$\lim_{t\to 0}\left[e^t\mathbf{i} + \frac{\sin t}{t}\mathbf{j} + e^{-t}\mathbf{k}\right] = \mathbf{i} + \mathbf{j} + \mathbf{k}$$
because
$$\lim_{t\to 0}\frac{\sin t}{t} = \lim_{t\to 0}\frac{\cos t}{1} = 1\text{L'Hopital's Rule}$$

3.

$$\begin{aligned}
\mathbf{r}(t) &= t\mathbf{i} - 3t\mathbf{j} + \tan t\mathbf{k} \\
\mathbf{r}'(t) &= \mathbf{i} - 3\mathbf{j} + \sec^2 t\mathbf{k} \neq 0 \\
&\mathbf{r} \text{ is smooth for all } t \neq \frac{\pi}{2} + n\pi = \frac{2n+1}{2}\pi \\
&\quad \text{Smooth on intervals of form } \left(-\frac{\pi}{2} + n\pi, \frac{\pi}{2} + n\pi\right) \\
&n \text{ is an integer.}
\end{aligned}$$

4. (a)

$$\int (e^t\mathbf{i} + \mathbf{j} + t\cos t\mathbf{k})dt = e^t\mathbf{i} + t\mathbf{j} + (\cos t + t\sin t)\mathbf{k} + \mathbf{C}$$

(b)

$$\begin{aligned}
&\int_0^{\frac{\pi}{4}} [(\sec t \tan t)\mathbf{i} + (\tan t)\mathbf{j} + (2\sin t\cos t)\mathbf{k}] \; dt \\
&= [\sec t\mathbf{i} + ln|\sec t|\mathbf{j} + \sin^2 t\mathbf{k}]_0^{\pi/4} \\
&= (\sqrt{2} - 1)\mathbf{i} + \ln\sqrt{2}\mathbf{j} + \frac{1}{2}\mathbf{k}
\end{aligned}$$