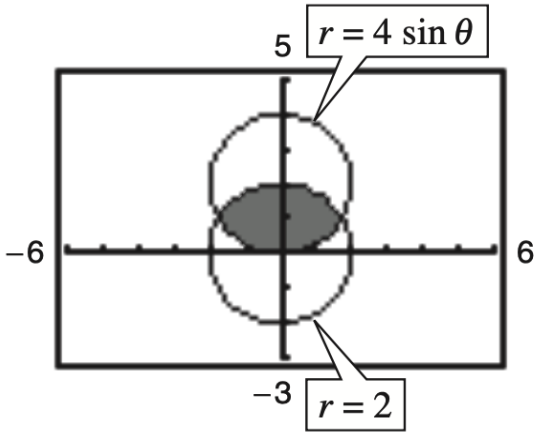
Assignment 6

1. Find the area of the given region analytically.
   (a) Common interior of $r = 4\sin\theta$ and $r = 2$
   (b) Interior of $r = 1 - \cos\theta$
   (c) Inner loop of $r = 2 - 4\cos\theta$
2. Convert the point from rectangular coordinates to spherical coordinates.
$$(-5, -5, \sqrt{2})$$
3. Find an equation in spherical coordinates for the surface represented by the rectangular equation.
$$x^2 + y^2 - 3z^2 = 0$$
4. Convert the point from spherical coordinates to cylindrical coordinates.
$$\left(10, \frac{\pi}{6}, \frac{\pi}{2}\right)$$

sol:

1. (a)
$$\begin{aligned} A &= \left[\frac{1}{2}\int_0^{\frac{\pi}{6}} (4\sin\theta)^2 d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2)^2 d\theta\right] \\ &= 16\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{\pi/6} + [4\theta]_{\pi/6}^{\pi/2} \\ &= \frac{8\pi}{3} - 2\sqrt{3} \end{aligned}$$



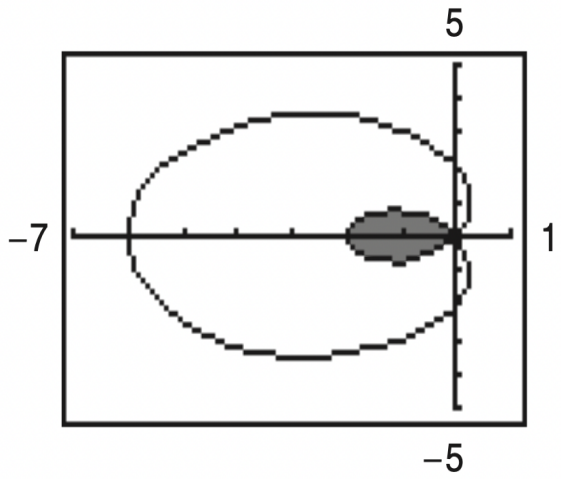
(b)

$$\begin{aligned}
A &= \frac{1}{2}\int_0^{2\pi} [1-\cos\theta]^2 \, d\theta \\
&= \frac{1}{2}\int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta) \, d\theta \\
&= \frac{1}{2}\int_0^{2\pi} \left(1 - 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) \, d\theta \\
&= \frac{1}{2}\left[\theta - 2\sin\theta + \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{2\pi} \\
&= \frac{1}{2}[2\pi + \pi] \\
&= \frac{3\pi}{2}
\end{aligned}$$

(c)

Half of the inner loop of $r = 2 - 4\cos\theta$ is traced out on the interval $0 \le \theta \le \dfrac{\pi}{3}$, so

$$\begin{aligned}
A &= 2 \cdot \frac{1}{2}\int_0^{\pi/3} (2 - 4\cos\theta)^2 \, d\theta \\
&= \int_0^{\pi/3} [4 - 16\cos\theta + 16\cos^2\theta] \, d\theta \\
&= \int_0^{\pi/3} [4 - 16\cos\theta + 8[1 + \cos 2\theta]] \, d\theta \\
&= [12\theta - 16\sin\theta + 4\sin 2\theta]_0^{\pi/3} \\
&= 12\left(\frac{\pi}{3}\right) - 16\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\sqrt{3}}{2}\right) \\
&= 4\pi - 6\sqrt{3}
\end{aligned}$$



2.

$$\rho = \sqrt{(-5)^2 + (-5)^2 + (\sqrt{2})^2} = \sqrt{52} = 2\sqrt{13}$$
$$\tan\theta = \frac{y}{x} = \frac{-5}{-5} = 1 \Rightarrow \theta = \frac{\pi}{4}$$
$$\phi = \arccos\frac{z}{\rho} = \arccos\frac{\sqrt{2}}{2\sqrt{13}} = \arccos\frac{\sqrt{26}}{26}$$
$$\left(2\sqrt{13}, \frac{\pi}{4}, \arccos\frac{\sqrt{26}}{26}\right)$$

3.

$$x^2 + y^2 - 3z^2 = 0, \text{ rectangular equation}$$
$$x^2 + y^2 + z^2 = 4z^2$$
$$\rho^2 = 4\rho^2\cos^2\phi$$
$$1 = 4\cos^2\phi$$
$$\cos\phi = \frac{1}{2}$$
$$\phi = \frac{1}{3}, \text{(cone) spherical equation}$$

4.

$$\left(10, \frac{\pi}{6}, \frac{\pi}{2}\right), \text{spherical}$$
$$r = 10\sin\frac{\pi}{2} = 10$$
$$\theta = \frac{\pi}{6}$$
$$z = 10\cos\frac{\pi}{2} = 0$$
$$\left(10, \frac{\pi}{6}, 0\right), \text{cylindrical}$$