Assignment 5

1. Determine the convergence or divergence of the sequence with the given nth term . If the sequence converges , find its limit.

$$a_n = \frac{\sin\sqrt{n}}{\sqrt{n}}$$

2. Determine the convergence or divergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^4}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^2-3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

3. Find the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{4^n(x-1)^n}{n}$$

4. Convert the rectangular equation to polar form and sketch its graph.

$$(x^2+y^2)^2 - 9(x^2-y^2) = 0$$

ans:

1.

$$\lim_{n\to\infty} \frac{\sin\sqrt{n}}{\sqrt{n}} = 0$$

Converges

2. (a)

$$\text{Let } f(x) = \frac{\ln x}{x^4}, f'(x) = \frac{1}{x^5} - \frac{4-\ln x}{x^3} < 0$$

f is positive, continuous, and decreasing for $x > 1$

$$\int_1^{\infty} x^{-4}\ln x\,dx = \lim_{b\to\infty}\left[\frac{-\ln x}{3x^3} - \frac{1}{9x^3}\right]_1^b = 0 + \frac{1}{9} = \frac{1}{9}$$

So, the series converges.

(b)

$$\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^2-3} \text{ converges by the Alternating Series Test.}$$

$$\lim_{n\to\infty} \frac{n}{n^2-3} = 0 \text{ and if}$$

$$f(x) = \frac{n}{n^2-3}, f'(x) = -\frac{(n^2+3)}{(n^2-3)^2} < 0$$

$\Rightarrow$ terms are decreasing.So, $a_{n+1} < a_n$

(c)

$$a_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} = \left(\frac{3}{2} \cdot \frac{5}{4} \cdots \frac{2n-1}{2n-2}\right) \frac{1}{2n} > \frac{1}{2n}$$

Because $\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series), so does the original series.

(d)

$$\begin{aligned}\lim_{n\to\infty} &= \lim_{n\to\infty} \left| \frac{n+1}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n} \right| \\ &= \lim_{n\to\infty} \left| \frac{e^{n^2}(n+1)}{e^{n^2+2n+1} n} \right| \\ &= \lim_{n\to\infty} \left(\frac{1}{e^{2n+1}}\right)\left(\frac{n+1}{n}\right) \\ &= (0)(1) = 0 < 1\end{aligned}$$

By the Ratio Test, the series converges.

3.

$$\begin{aligned}\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| &= \lim_{n\to\infty} \left| \frac{4^{n+1}(x-1)^{n+1}/(n+1)}{4^n (x-1)^n / n} \right| \\ &= \lim_{n\to\infty} \left| 4(x-1) \frac{n}{n+1} \right| \\ &= |4(x-1)|\end{aligned}$$

$$R = \frac{1}{4}, \text{ Center} : 1$$

Because the series converges when $x = \frac{3}{4}$ and diverges when $x = \frac{5}{4}$, the interval of convergence is $\left[\frac{3}{4}, \frac{5}{4}\right)$

4.

$$
\begin{aligned}
(x^2+y^2)^2 - 9(x^2-y^2) &= 0 \\
(r^2)^2 - 9(r^2\cos^2\theta - r^2\sin^2\theta) &= 0 \\
r^2[r^2 - 9(\cos 2\theta)] &= 0 \\
r^2 &= 9\cos 2\theta
\end{aligned}
$$