Assignment 4

1. Use the definition of Taylor series to find the Taylor series, centered at c, for the function.

$$f(x) = \ln x \quad , \quad c = 1$$

2. Find all points (if any) of horizontal and vertical tangency to the curve.

$$x = \cos\theta \ , \ y = 2\sin 2\theta$$

3. Determine the open t-intervals on which the curve is concave downward or concave upward.

$$x = 4\cos t \ , \ y = 2\sin t \ , \ 0 < t < 2\pi$$

4. Find the area of the surface generated by revolving the curve about each given axis.

$$x = \frac{t^3}{3} \ , \ y = t + 1 \ , \ 1 \le t \le 2 \ , \ \text{y-axis}$$

sol:

1.

$$\begin{aligned}
&\text{For } c = 1, \text{ you have,} \\
f(x) &= \ln x \ , \ f(1) = 0 \\
f'(x) &= \frac{1}{x} \ , \ f'(1) = 1 \\
f''(x) &= -\frac{1}{x^2} \ , \ f''(1) = -1 \\
f'''(x) &= \frac{2}{x^3} \ , \ f'''(1) = 2 \\
f^{(4)}(x) &= -\frac{6}{x^4} \ , \ f^{(4)}(1) = -6 \\
f^{(5)}(x) &= \frac{24}{x^5} \ , \ f^{(5)}(1) = 24 \\
&\text{and so on.Therefore,you have:} \\
\ln x &= \sum_{n=0}^{\infty} \frac{f^{(n)}(1)(x-1)^n}{n!} \\
= 0 + (x-1) - \frac{(x-1)^2}{2!} &+ \frac{2(x-1)^3}{3!} - \frac{6(x-1)^4}{4!} + \frac{24(x-1)^5}{5!} - \cdots \\
= (x-1) - \frac{(x-1)^2}{2} &+ \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} - \cdots \\
&= \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}
\end{aligned}$$

2.

$$x = \cos\theta, y = 2\sin 2\theta$$
Horizontal tangents: $\dfrac{dy}{d\theta} = 4\cos 2\theta = 0$ when $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$.

Points: $\left(\dfrac{\sqrt{2}}{2}, 2\right), \left(-\dfrac{\sqrt{2}}{2}, -2\right), \left(-\dfrac{\sqrt{2}}{2}, 2\right), \left(\dfrac{\sqrt{2}}{2}, -2\right)$

Vertical tangents:$\dfrac{dx}{d\theta} = -\sin\theta = 0$ when $\theta = 0, \pi$

Points: $(1, 0)(-1, 0)$

3.

$$\begin{aligned} x = 4\cos t, y = 2\sin t, 0 < t < 2\pi \\ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-4\sin t} = -\frac{1}{2}\cot t \\ \frac{d^2y}{dx^2} = \frac{d/dt[-1/2\cot t]}{dx/dt} = \frac{1/2\csc^2 t}{-4\sin t} = \frac{-1}{8\sin^3 t} \\ \text{Concave upward on } \pi < t < 2\pi \\ \text{Concave downward on } 0 < t < \pi \end{aligned}$$

4.

$$\begin{aligned} x &= \frac{1}{3}t^3, y = t + 1, 1 \le t \le 2, \text{y-axis} \\ \frac{dx}{dt} &= t^2, \frac{dy}{dt} = 1 \\ S &= 2\pi \int_1^2 \frac{1}{3}t^3\sqrt{t^4 + 1}\ dt \\ &= \frac{\pi}{9}\left[(x^4 + 1)^{3/2}\right]_1^2 \\ &= \frac{\pi}{9}(17^{3/2} - 2^{3/2}) \\ &\approx 23.48 \end{aligned}$$