Assignment 3

1. Find the nth Maclaurin polynomial for the function.

$$f(x) = \frac{1}{1-x} \quad , \quad n = 5$$

2. Find the nth Taylor polynomial for the function, centered at c.

$$f(x) = x^2 \cos x \quad , \quad n = 2 \quad , \quad c = \pi$$

3. Find the interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(x-3)^{n+1}}{(n+1)4^{n+1}}$$

4. Use the power series $\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n \quad , \quad |x| < 1$ to find a power series for the function, centered at 0, and determine the interval of convergence.

$$f(x) = \ln(x^2+1)$$

sol:

1.

$$f(x) = \frac{1}{1-x} \ , \ f(0) = 1$$
$$f'(x) = \frac{1}{(1-x)^2} \ , \ f'(0) = 1$$
$$f''(x) = \frac{2}{(1-x)^3} \ , \ f''(0) = 2$$
$$f'''(x) = \frac{6}{(1-x)^4} \ , \ f'''(0) = 6$$
$$f^{(4)}(x) = \frac{24}{(1-x)^5} \ , \ f^{(4)}(0) = 24$$
$$f^{(5)}(x) = \frac{120}{(1-x)^6} \ , \ f^{(5)}(0) = 120$$
$$P_5(x) = 1 + x + \frac{2x^2}{2!} + \frac{6x^3}{3!} + \frac{24x^4}{4!} + \frac{120x^5}{5!} = 1 + x + x^2 + x^3 + x^4 + x^5$$

2.

$$f(x) = x^2 \cos x \ , \ f(\pi) = -\pi^2$$
$$f'(x) = \cos x - x^2 \sin x \ , \ f'(\pi) = -2\pi$$
$$f''(x) = 2\cos x - 4x \sin x - x^2 2 \cos x \ , \ f''(\pi) = -2 + \pi^2$$
$$P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(\pi^2-2)}{2}(x-\pi)^2$$

3.

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^{n+2/[(n+2)4^{n+2}]}}{(x-3)^{n+1/[(n+1)4^{n+1}]}}\right| = \lim_{n\to\infty}\left|\frac{(x-3)(n+1)}{4(n+2)}\right| = \lim_{n\to\infty}\left|\frac{x-3}{4}\right|$$

$R = 4$

Interval : $\quad -1 < x < 7$

When $x = 7$, $\sum_{n=0}^{\infty}\frac{4^{n+1}}{(n+1)4^{n+1}} = \sum_{n=0}^{\infty}\frac{1}{n+1}$ diverges

When $x = -1$, $\sum_{n=0}^{\infty}\frac{(-4)^{n+1}}{(n+1)4^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}$ converges

Therefore,the interval of convergence is $[-1, 7)$

4.

$$\frac{2x}{x^2+1} = 2x\sum_{n=0}^{\infty}(-1)^n x^{2n} = \sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}$$

Because $\frac{d}{dx}(\ln(x^2+1)) = \frac{2x}{x^2+1}$, you have

$$\ln(x^2+1) = \int\left[\sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}\right]dx = C + \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{n+1}\ ,\ -1 \le x \le 1$$

To solve for C,let $x = 0$ and conclude that $C = 0$.Therefore,

$$\ln(x^2+1) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{n+1}\ ,\ [-1, 1]$$