Assignment 2

1. Use the Direct Comparison test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{(\cos n) + 2}{\sqrt{n}}$$

2. Use the Limit Comparison test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$$

3. Determine the convergence or divergence of the series.

   (a)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$$

   (b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

4. Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^3}$$

5. Use the Root Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$$

sol:

1.
$$\frac{(\cos n) + 2}{\sqrt{n}} \geq \frac{1}{\sqrt{n}} \text{ for } n \geq 1$$

Therefore , $\displaystyle\sum_{n=1}^{\infty} \frac{(\cos n) + 2}{\sqrt{n}}$ diverges by comparison with the divergent p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

2.
$$\lim_{n\to\infty} \frac{n/[(n+1)2^{n-1}]}{1/(2^{n-1})} = \lim_{n\to\infty} \frac{n}{n+1} = 1$$

Therefore , $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$ converges by a limit comparison

with the convergent geometric series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$

3. (a)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt[3]{n}}$$

$$\lim_{n\to\infty} \frac{n^{1/2}}{n^{1/3}} = \lim_{n\to\infty} n^{1/6} = \infty$$

Diverges by the nth-Term Test

(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

$$a_{n+1} = \frac{1}{(2n+3)!} < \frac{1}{(2n+1)!} = a_n$$

$$\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$$

Converges by Theorem 9.14

4.

$$\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^3}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{6^{n+1}/(n+2)^3}{6^n/(n+1)^3}\right|$$

$$= \lim_{n\to\infty} \frac{6(n+1)^3}{(n+2)^3} = 6 > 1$$

Therefore, the series diverges by the Ratio Test.

5.

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2} = \sum_{n=1}^{\infty} \frac{(n!)^n}{(n^2)^n}$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{(n!)}{(n^2)^n}} = \lim_{n\to\infty} \frac{n!}{n^2} = \infty$$

Therefore, by the Root Test, the series diverges.