Assignment 1

1. Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

$$a_n = \frac{\ln(n^3)}{2n}$$

2. Find the sum of the convergent series.

$$\sum_{n=0}^{\infty} [(0.3)^n + (0.8)^n]$$

3. Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$$

4. Use the Integral Test to determine the convergence or divergence of the series.

$$\frac{\ln 2}{\sqrt{2}} + \frac{\ln 3}{\sqrt{3}} + \frac{\ln 4}{\sqrt{4}} + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 6}{\sqrt{6}} + \dots$$

sol:

1.

$$\begin{aligned}\lim_{n\to\infty} \frac{\ln(n^3)}{2n} &= \lim_{n\to\infty} \frac{3}{2}\frac{\ln(n)}{n} \\ &= \lim_{n\to\infty} \frac{3}{2}\left(\frac{1}{n}\right) \\ &= 0 \quad \text{,converges}\end{aligned}$$

2.

$$\begin{aligned}\sum_{n=0}^{\infty} [(0.3)^n + (0.8)^n] &= \sum_{n=0}^{\infty} \left(\frac{3}{10}\right)^n + \sum_{n=0}^{\infty} \left(\frac{8}{10}\right)^n \\ &= \frac{1}{1-(3/10)} + \frac{1}{1-(8/10)} \\ &= \frac{10}{7} + 5 \\ &= \frac{45}{7}\end{aligned}$$

3.

$$\begin{aligned}S_n &= \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \dots + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right) \\ &= 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\end{aligned}$$

$$\begin{aligned}\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right) &= \lim_{n\to\infty} S_n = \lim_{n\to\infty} 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \\ &= \frac{3}{2} \quad \text{,converges}\end{aligned}$$

4.

$$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$$

Let, $f(x) = \dfrac{\ln x}{\sqrt{x}}, f'(x) = \dfrac{2 - \ln x}{2x^{3/2}}$

f is positive,continuous,and decreasing for $x > e^2 \approx 7.4$

$$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}\, dx = \left[2\sqrt{x}(\ln x - 2)\right]_2^{\infty} = \infty$$

So,the series diverges.