Assignment 12

1. Evaluate the iterated integral.

   (a) $\int_1^3 \int_0^y \frac{4}{x^2+y^2}\, dxdy$ (c) $\int_0^{\ln(10)} \int_{e^x}^{10} \frac{1}{\ln y}\, dydx$

   (b) $\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x\, dxdy$ (d) $\int\int_R -2y\, dA\ ,\ R:\ y = 4-x^2, y = 4-x$

2. Change the order of integration.

$$\int_{-1}^{2}\int_0^{e^{-x}} f(x,y)\, dydx$$

3. Find the average value of $f(x,y)$ over the plane region R.

$$f(x,y) = e^{x+y}, R : \text{triangle with vertices } (0,0),(0,1),(1,1)$$

sol:

1. (a)

$$\begin{aligned}
\int_1^3 \int_0^y \frac{4}{x^2+y^2}\, dxdy &= \int_1^3 \left[\frac{4}{y}\arctan\left(\frac{x}{y}\right)\right]_0^y dy \\
&= \int_1^3 \frac{4}{y}\left(\frac{\pi}{4}\right) dy \\
&= \int_1^3 \frac{\pi}{4}\, dy \\
&= [\pi \ln y]_1^3 \\
&= \pi \ln 3
\end{aligned}$$

   (b)

$$\begin{aligned}
\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x\, dxdy &= \int_0^4 \int_0^{\sqrt{x}} \sqrt{x}\sin x\, dydx \\
&= \int_0^4 \left[y\sqrt{x}\sin x\right]_0^{\sqrt{x}} dx \\
&= \int_0^4 x\sin x\, dx \\
&= [\sin x - x\cos x]_0^4 \\
&= 1.858
\end{aligned}$$

   (c)

$$\begin{aligned}
\int_0^{\ln(10)} \int_{e^x}^{10} \frac{1}{\ln y}\, dydx &= \int_1^{10}\int_0^{\ln y} \frac{1}{\ln y}\, dxdy \\
&= \int_1^{10} \left[\frac{x}{\ln y}\right]_0^{\ln y} dy \\
&= \int_1^{10} dy \\
&= [y]_1^{10} \\
&= 9
\end{aligned}$$

(d)

$$\begin{aligned}
\int_3^4 \int_{4-y}^{\sqrt{4-y}} -2y\ dxdy &= \int_0^1 \int_{4-x}^{4-x^2} -2y\ dydx \\
&= \int_0^1 \left[-y^2\right]_{4-x}^{4-x^2}\ dx \\
&= -\int_0^1 \left[(4-x^2)^2 - (4-x)^2\right]\ dx \\
&= -\int_0^1 [16 - 8x^2 + x^4 - (16 - 8x + x^2)]\ dx \\
&= -\left[-3x^3 + \frac{x^5}{5} + 4x^2\right]_0^1 \\
&= -\frac{6}{5}
\end{aligned}$$

2.

$$\begin{aligned}
&\int_{-1}^2 \int_0^{e^{-x}} f(x,y)\ dydx\ ,\ 0 \le y \le e^{-x}\ ,\ -1 \le x \le 2 \\
&\quad = \int_0^{e^{-2}} \int_{-1}^2 f(x,y)\ dxdy\ +\ \int_{e^{-2}}^e \int_{-1}^{-\ln y} f(x,y)\ dxdy
\end{aligned}$$

3.

$$\begin{aligned}
A &= \frac{1}{1/2} \int_0^1 \int_x^1 e^{x+y}\ dydx = 2\int_0^1 e^{x+1} - e^{2x}\ dx \\
&= 2\left[e^{x+1} - \frac{1}{2}e^{2x}\right]_0^1 \\
&= 2\left[e^2 - \frac{1}{2}e^2 - e + \frac{1}{2}\right] \\
&= e^2 - 2e + 1 \\
&= (e-1)^2
\end{aligned}$$