Assignment 11

1. Find a set of parametric equations for the tangent line to the curve of intersection of the surfaces at the given point.

$$z = \sqrt{x^2+y^2},\ 5x-2y+3z=22,\ (3,4,5)$$

2. Find all relative extrema and saddle points of the function.

(a) $f(x,y) = -5x^2+4xy-y^2+16x+10$ (b) $f(x,y) = \sqrt[3]{x^2+y^2}+2$

3. Use Lagrange multipliers to find the indicated extrema, assuming that x, y, and z are positive.

$$\text{Maximize : } f(x,y,z) = x+y+z$$
$$\text{Constraint : } x^2+y^2+z^2=1$$

4. Use Lagrange multipliers to find the indicated extrema of f subject to two constraints, assuming that x, y, and z are nonnegative.

$$\text{Minimize : } f(x,y,z) = x^2+y^2+z^2$$
$$\text{Constraint : } x+2z=6,\ x+y=12$$

sol:

1.

$$\begin{aligned}
F(x,y,z) &= \sqrt{x^2+y^2}-z\\
\nabla F(x,y,z) &= \frac{x}{\sqrt{x^2+y^2}}\mathbf{i}+\frac{y}{\sqrt{x^2+y^2}}\mathbf{j}-\mathbf{k}\\
\nabla F(3,4,5) &= \frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}-\mathbf{k}\\
G(x,y,z) &= 5x-2y+3z-22\\
\nabla G(x,y,z) &= 5\mathbf{i}-2\mathbf{j}+3\mathbf{k}\\
\nabla G(3,4,5) &= 5\mathbf{i}-2\mathbf{j}+3\mathbf{k}\\
\nabla F\times\nabla G &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \frac{3}{5} & \frac{4}{5} & -1\\ 5 & -2 & 3\end{vmatrix} = \frac{2}{5}\mathbf{i}-\frac{34}{5}\mathbf{j}-\frac{26}{5}\mathbf{k}
\end{aligned}$$

Direction numbers: $1, -17, -13$

$$x = 3+t,\ y = 4-17t,\ z = 5-13t$$

2. (a)

$$f_x = -10x+4y+16 = 0$$
$$f_y = 4x-2y = 0$$

Solving simultaneously yields $x=8, y=16$

$$f_{xx} = -10,\ f_{yy} = -2,\ f_{xy} = 4$$

At the critical point $(8,16)$, $f_{xx}<0$ and $f_{xx}f_{yy}-(f_{xy})^2>0$

So, $(8,16,74)$ is a relative maximum

(b)

$$f_x = \frac{2x}{3(x^2+y^2)^{2/3}} = 0$$
$$f_y = \frac{2y}{3(x^2+y^2)^{2/3}} = 0$$
$$x = 0, y = 0$$

Because $f(x,y) \geq 2$ for all $(x,y), (0,0,2)$ is a relative minimum

3.

$$1 = \lambda 2x$$
$$1 = \lambda 2y$$
$$1 = \lambda 2z$$
$$x = y = z = \frac{1}{2\lambda}$$
$$x^2 + y^2 + z^2 = \frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = \frac{3}{4\lambda^2} = 1$$
$$\lambda^2 = \frac{3}{4} \Rightarrow \lambda = \frac{\sqrt{3}}{2} \Rightarrow x = y = z = \frac{1}{\sqrt{3}}$$
$$f(x,y,z) = \frac{3}{\sqrt{3}} = \sqrt{3}$$

4.

$$\nabla f = \lambda \nabla g + \mu \nabla h$$
$$2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k} = \lambda(\mathbf{i+2k}) + \mu(\mathbf{i} + \mathbf{j})$$
$$2x = \lambda + \mu$$
$$2y = \mu$$
$$2z = 2\lambda$$
$$2x = 2y + z$$
$$x + 2z = 6 \Rightarrow z = \frac{6-x}{2} = 3 - \frac{x}{2}$$
$$x + y = 12 \Rightarrow y = 12 - x$$
$$2x = 2(12-x) + \left(\frac{x}{2}\right) \Rightarrow \frac{9}{2}x = 27 \Rightarrow x = 6$$
$$x = 6, z = 0$$
$$f(6,6,0) = 72$$