1. Find the following limit. (If the limit does not exist or has an infinite limit, you should point it out. In addition, also remember the definition of definite integral). (15%)

   (a) $\lim_{n\to\infty} \ln(\frac{1}{n}\times\frac{2}{n}\ldots\times\frac{n}{n})^{\frac{1}{n}}$

   (b) $\lim_{x\to 0}\frac{1}{x}\int_0^x \cos(4t)dt$

   (c) $\lim_{x\to 0}\cot(x)-\frac{1}{x}$

   (d) $\lim_{x\to 0}\frac{\arctan(\sin(2x))}{\tan(\arcsin(3x))}$

   (e) $\lim_{x\to\infty}(1+\frac{1}{x})^{4x}$

2. Find the point $a > 0$ satisfying (8%)

$$\int_1^a \frac{2}{t}dt = \int_a^{\frac{1}{8}}\frac{1}{t}dt$$

3. Assume the inverse function of $f(x) = x^5 + 2x^3 + x - 2$ is $g(x)$, find $g'(f(1))$ (5%).

4. Evaluate the following integral. (12%)

   (a) $\int_{\sqrt{3}}^{3}\frac{1}{x\sqrt{4x^2-9}}dx$?

   (b) $\int \frac{2}{\sqrt{-x^2+4x}}dx$

   (c) $\int \frac{5^{2x}}{1+5^{2x}}dx$

   (d) $\int_{\frac{1}{4}}^{\frac{1}{2}}\frac{arcsin(\sqrt{x})}{\sqrt{x(1-x)}}dx$

5. Find the equation of the tangent line $y = \log_{10}(3x)$ at $x = 5$. (5%)

6. Evaluate the following integral. (16%)

(a) $\int (\ln x)^3 dx$

(b) $\int e^x cosx \ dx$

(c) $\int_{-\pi}^{\pi} \sin(mx) cos(nx) dx$, where m and n are positive integers

(d) $\int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}} dx$

7. Evaluate the following integral. (16%)

(a) $\int \frac{sec^2(x)}{(tanx)(tanx+1)}$ dx

(b) $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}$ dx

(c) $\int_{-\infty}^{0} xe^x \ dx$

(d) $\int_0^3 \frac{1}{\sqrt[3]{x-1}} dx$

8. Determine whether the following integral diverges or converges. (9%)

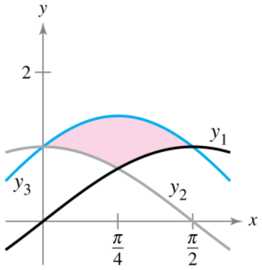
(a) $\int_0^1 \frac{1}{\sqrt[7]{x}} dx$

(b) $\int_1^{\infty} \frac{sin^2 x}{x^2} dx$

(c) $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$

9. Find the area of the given region bounded by the graphs y_1, y_2 and y_3 (The pink region in the following figure). (6%)

$$y_1 = sinx, y_2 = cosx, y_3 = cosx + sinx$$



10. Find the volume of the solid generated by revolving the region bounded by the graph of f(x) $= e^{-x}$ and the x-axis (0≤ x ≤ ln 2) about the line $y = -1$. (8%)